Resolution of constraints and gauge equivalence in algebraic Schrödinger representation of quantum electrodynamics

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The algebraic formalism of QED is expounded in order to demonstrate both the resolution of constraints and to verify gauge equivalence between temporal gauge and Coulomb gauge on the quantum level. In the algebraic approach energy eigenstates of QED in temporal gauge are represented in an algebraic GNS basis. The corresponding Hilbert space is mapped into a functional space of generating functional states. The image of the QED-Heisenberg dynamics becomes a functional energy equation for these states. In the same manner the Gauß constraint is mapped into functional space. By suitable transformations the functional image of the Coulomb forces is recovered in temporal gauge. The equivalence of this result with the functional version of QED in Coulomb gauge is demonstrated. The meaning of the various transformations and their relations are illustrated for the case of harmonic oscillators. If applied to QCD this method allows an exact derivation of effective color "Coulomb" forces, in addition it implies a clear conception for the incorporation of various algebraic representations into the formal Heisenberg dynamics and establishes the algebraic "Schrödinger" equation for quantum fields in functional space.

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1. Introduction

Quantum field theories describe physical systems with an infinite number of degrees of freedom. As a consequence these theories possess anti-liminary field algebras which have an infinite number of inequivalent representations [1]. This fact necessitates to abandon the methods of Hilbert space quantum mechanics in favour of an application of algebraic methods for a proper evaluation of the theory. So far these algebraic methods have been successfully used for the derivation of definite physical results only in the case of nonrelativistic quantum field theories [2], but not in the case of relativistic quantum field theories [3]. Due to local interactions, singularities, renormalization, etc., the latter ones are at present not in the realm of rigorous mathematical deduction for nontrivial realistic $d = 4$ models. Nevertheless, even for such systems one can try to extract valuable physical information by means of algebraic concepts if in a first step one does it without proofs of existence. In this case a formalism can be developed which is adapted to the needs of relativistic quantum field theories and is based on the central concepts of algebraic representation theory. This formalism can be characterized as functional quantum theory and was introduced by Stumpf and coworkers, see [4]. It was successfully applied to the problem of deriving effective theories for composite particles in relativistic quantum field theory.

In the present paper and in a subsequent paper the application of functional quantum theory will be extended in order to include the constraints of abelian and nonabelian gauge theories into the algebraic formalism and to draw conclusions about quantization, effective forces, etc., resulting from the incorporation or elimination of constraints. The central ingredients of this approach are Heisenberg's equation of motion and the GNS-state construction [5]; and apart from the problem of existence, all formal proofs are based on exact algebraic rearrangements. Furthermore, an appropriate modern treatment of the equations of motion which is in accordance with algebraic representation theory, consists in a map of the corresponding state space into a functional state space, mapping the Heisenberg equations of motion into functional equations. From the very beginning we will use this formulation which extends the usual functional calculus of statistical mechanics and quantum field theory by

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explicit construction of linear functional state spaces isomorphic to the underlying state spaces of the quantum fields, and which motivates the term 'functional quantum theory'. Details of this method are given in [4].

However, for the resolution of the constraints in quantized gauge theories only a part of the formalism of functional quantum theory is needed. Leaving aside the treatment of the metrical structure, i.e., the inner products of functional states and the construction of the time evolution operator (effective Hamiltonian), both depending on the special algebraic representation, we concentrate our treatment on that part of the functional quantum theory which is independent of special representations and which on the other hand is sufficient to solve the constraint problem. This includes that the Hamiltonian occurring in Heisenberg's equation of motion is only a formal quantity connected with the representation-free dynamics of the system which is unobservable. The effective time evolution of the system results from the combination of the formal system dynamics with special representations and yields an effective generator of time translations in functional space different from H. In addition, in our approach the physical states and not the V.E.V. are the primary objects of discussion. This difference to common treatments, where V.E.V.s are exclusively considered, and the omission of inner product construction and effective time evolution have to be observed in the following in order to avoid misinterpretations of the calculations presented in this paper.

The use of Heisenberg's equations of motion is closely related to the Hamilton formalism and in gauge theories this formalism is connected with the temporal gauge if one wants to quantize the field in "cartesian" field coordinates [6]. In this way the difficulties with curvilinear coordinates, etc., resulting from other gauges, are avoided. Hence we will work in a noncovariant gauge. Although in relativistic quantum field theories the covariant gauges play a preferred role, for theoretical as well as practical reasons also noncovariant gauges are increasingly studied [7]. This stems from the fact that in spite of great efforts the quantization of abelian and nonabelian gauge fields is still controversial. For instance, in [8] it is stated that for field quantization the temporal gauge seems to be the only practical way to proceed, while in [9] it is stated that the temporal gauge is still the most complicated and cumbersone gauge choice. This indicates that at present a compelling rigorous deduction of gauge field quantization does not exist. Indeed, in particular for nonabelian gauge fields in noncovariant gauges the consequences of field quantization have so far not been rigorously evaluated. With respect to covariant gauges the quantization enforces the introduction of auxiliary fields, the consequences of which are also unclear. In order to avoid complications by auxiliary fields and in addition by indefinite metric the noncovariant gauges are indicated.

Among these noncovariant gauges the Coulomb gauge was the object of many investigations. However, in Coulomb gauge a nonabelian field is described in generalized noncartesian field coordinates and field momenta [6] which prevents a straightforward application of the algebraic field quantization method and enforces a quantum field description by a mechanical Hamilton operator with an infinite number of degrees of freedom and incalculable noncartesian metric. Although in quantum electrodynamics the Coulomb gauge is the most familiar and the most "physical" noncovariant gauge, for nonabelian fields we have to reject it in order to preserve the applicability of algebraic methods. With respect to these methods the temporal gauge seems to be the most appropriate one. We will discuss it in these papers for abelian as well as for nonabelian fields.

We start in this first paper with the treatment of quantum electrodynamics. By means of the algebraic formalism we transform the functional version of the temporal gauge into a form in which the Coulomb forces explicitly appear. This is equivalent to the elimination of the Gauß constraint and its redundant variables. In the following paper we will demonstrate that also nonabelian quantum fields in temporal gauge can be treated along the same lines, thus avoiding the difficulty of introducing noncartesian coordinates into the theory. It is this advantage which is the essential part of the algebraic approach and although quantum electrodynamics has been so extensively treated (even in this case) the scarcely used algebraic approach offers new insights into the structure of abelian gauge theories. In this respect a comment is given at the end of Section 6.

2. Classical spinor electrodynamics

As will be demonstrated in the following, in spite of the fact that all gauges should be equivalent, the temporal gauge is a distinguished gauge for the application of algebraic representation theory to quantum

fields. On the other hand the temporal gauge still allows residual gauge transformations which are accompanied by the appearance of redundant variables. To make the theory well defined these redundant variables must be eliminated. In electrodynamics the Coulomb gauge satisfies this requirement. Hence one is confronted with the task to derive the Coulomb force in temporal gauge. We first treat the temporal gauge for classical electromagnetic fields. We adopt the metric $g_{\mu\lambda} = \mathrm{diag}\,(1,-1,-1,-1)$ with $x^\mu = (t, \boldsymbol{r})$ and $A^\mu = (A^0, \boldsymbol{A})$ in the following.

We take into account the coupling of the electromagnetic field to fermions. The Lagrangian of this system reads

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \tag{1}$$

with $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu := \partial_\mu - ieA_\mu$. The canonical conjugate momenta of the electromagnetic field are

$$\begin{aligned} \pi^k &:= \frac{\delta\mathcal{L}}{\delta\partial^0 A_k} = F_{0k} = E^k,\ k = 1,2,3, \\ \pi^0 &:= \frac{\delta\mathcal{L}}{\delta\partial^0 A_0} = 0. \end{aligned} \tag{2}$$

The vanishing of π_0 indicates that A^0 is a superfluous variable. We use the gauge freedom to remove A^0 from the theory by setting $A_0 \equiv 0$, see, e.g., [8]. In this temporal gauge we formulate the field equations resulting from (1) in terms of canonical conjugate variables and for a compact notation we introduce superspinors $\psi := \{\psi, \psi^c\}$, with ψ^c charge conjugated spinor. Then the field equations read

$$\begin{aligned} i\dot{\psi} &= -(i\alpha^k\partial_k - \beta m)\psi - e\gamma^0\gamma^k A_k\sigma^3\psi, \\ \dot{E}^k &= i\frac{e}{2}\psi C\gamma^k\sigma^2\psi + (\partial_j\partial^k A^j - \partial_j\partial^j A^k), \\ \dot{A}^k &= -E^k, \\ \partial_k E^k &= -i\frac{e}{2}\psi C\gamma^0\sigma^2\psi, \end{aligned} \tag{3, 4}$$

where the σ-matrices ($\equiv$ Pauli matrices) act on the superspinor indices. The Hamiltonian belonging to this system is given by

$$\begin{aligned} H = &\frac{1}{2}\int \mathrm{d}^3r[E_kE_k + (\boldsymbol{\nabla}\times\boldsymbol{A})_k(\boldsymbol{\nabla}\times\boldsymbol{A})_k] \\ &+ \int \mathrm{d}^3r\psi^+\left(\frac{1}{i}\vec{\alpha}\cdot\boldsymbol{D} + m\beta\right)\psi \end{aligned} \tag{5}$$

if formulated in terms of conjugate variables.

If instead of the Lagrange formalism one applies the Hamilton formalism for the derivation of the field equations, only equations (3) result but not the Gauß law (4). This somewhat unsatisfactory fact is, however, no shortcoming of the Hamilton formalism, because H allows residual gauge transformations which are closely connected with the Gauß law, as can be best demonstrated in the quantum version of the theory.

The residual gauge transformations are defined by

$$\psi'(\boldsymbol{r}, t) = \exp\,[-ie\omega(\boldsymbol{r})]\psi(\boldsymbol{r}, t) \tag{6}$$

for Dirac spinors, and

$$\begin{aligned} \boldsymbol{A}'(\boldsymbol{r}, t) &= \boldsymbol{A}(\boldsymbol{r}, t) + \boldsymbol{\nabla}\omega(\boldsymbol{r}), \\ \boldsymbol{E}'(\boldsymbol{r}, t) &= \boldsymbol{E}(\boldsymbol{r}, t) \end{aligned} \tag{7}$$

for the fields and leave H invariant. The gauge functions $\omega(\boldsymbol{r})$ are time-independent. This invariance indicates the existence of redundant variables. In the classical version of the theory we can transform the system into the Coulomb gauge. We remove the longitudinal part of $\boldsymbol{A}(\boldsymbol{r}, 0)$ from the Hamiltonian (5) taken at $t = 0$. To that end we decompose the fields into transversal and longitudinal parts:

$$\boldsymbol{A} = \boldsymbol{A}_{\mathrm{tr}} + \boldsymbol{A}_\ell;\ \boldsymbol{E} = \boldsymbol{E}_{\mathrm{tr}} + \boldsymbol{E}_\ell \tag{8}$$

In this decomposition the dynamical equations (3) can be written

$$\begin{aligned} i\dot{\psi} = &-(i\alpha^k\partial_k - \beta m)\psi - e\gamma^0\gamma^k A^{\mathrm{tr}}_k\sigma^3\psi \\ &- e\gamma^0\gamma^k A^\ell_k\sigma^3\psi, \\ i\dot{E}^k_{\mathrm{tr}} = &-\frac{e}{2}P_{\mathrm{tr}}\psi C\gamma^k\sigma^2\psi + i\Delta A^k_{\mathrm{tr}}, \\ i\dot{E}^k_\ell = &-\frac{e}{2}P_\ell\psi C\gamma^k\sigma^2\psi, \\ \dot{A}^k_{\mathrm{tr}} = &-E^k_{\mathrm{tr}};\ \dot{A}^k_\ell = -E^k_\ell, \end{aligned} \tag{9}$$

where P_{tr} and P_ℓ are the corresponding projection operators, and the Hamiltonian (5) can be written as

$$\begin{aligned} H = &\frac{1}{2}\int \mathrm{d}^3r\left[E^k_{\mathrm{tr}}E^k_{\mathrm{tr}} + (\boldsymbol{\nabla}\times\boldsymbol{A}_{\mathrm{tr}})_k(\boldsymbol{\nabla}\times\boldsymbol{A}_{\mathrm{tr}})_k\right] \\ &+\frac{1}{2}\int \mathrm{d}^3r E^k_\ell E^k_\ell \\ &+\int \mathrm{d}^3r\psi^+\left(\frac{1}{i}\vec{\alpha}\cdot\boldsymbol{\nabla} + m\beta\right)\psi \\ &-\int \mathrm{d}^3r\psi^+e\vec{\alpha}\cdot(\boldsymbol{A}_{\mathrm{tr}} + \boldsymbol{A}_\ell)\psi. \end{aligned} \tag{10}$$

As H is gauge invariant against the transformations (6) and (7), it is for any gauge $H = H'$. We use this property to remove superfluous variables. If we decompose the A-equation of (7) into longitudinal and transversal parts we obtain

$$\boldsymbol{A}'_{\mathrm{tr}} = \boldsymbol{A}_{\mathrm{tr}}; \quad \boldsymbol{A}'_{\ell} = \boldsymbol{A}_{\ell} + \boldsymbol{\nabla}\omega \tag{11}$$

Furthermore, the longitudinal part of $\boldsymbol{A}$ has to be expressible as a gradient $\boldsymbol{A}_{\ell} = \boldsymbol{\nabla}\lambda$. Hence we can write

$$\boldsymbol{\nabla}\lambda' = \boldsymbol{\nabla}\lambda + \boldsymbol{\nabla}\omega \tag{12}$$

Gauge fixing is achieved by postulating $\boldsymbol{A}'_{\ell} \equiv 0$. Then (12) goes over into

$$\boldsymbol{\nabla}\lambda = -\boldsymbol{\nabla}\omega, \tag{13}$$

the solution of which is $\omega = -\lambda$, apart from a constant. In this case H' reads

$$\begin{aligned} H' = H^{\mathrm{el}}_{\mathrm{tr}} &+ \frac{1}{2}\int \mathrm{d}^3 r E^k_{\ell} E^k_{\ell} \\ &+ \int \mathrm{d}^3 r \psi^{+}\left(\frac{1}{i}\vec{\alpha}\cdot\boldsymbol{\nabla} + m\beta\right)\psi \\ &- \int \mathrm{d}^3 r \psi^{+} e\vec{\alpha}\cdot\boldsymbol{A}_{\mathrm{tr}}\psi, \end{aligned} \tag{14}$$

where in addition the Gauß law

$$\partial_k E^k_{\ell} = -i\frac{e}{2}\psi C\gamma^0\sigma^2\psi \tag{15}$$

has to be taken into account. With the ansatz $E^k_{\ell} = -\partial^k\varphi$ we obtain from (15)

$$\varphi(\boldsymbol{r}) = \frac{ie}{8\pi}\int \frac{\mathrm{d}^3 r'}{|\boldsymbol{r}-\boldsymbol{r}'|}(\psi C\gamma^0\sigma^2\psi)(\boldsymbol{r}'), \tag{16}$$

and by partial integration we have

$$\frac{1}{2}\int \mathrm{d}^3 r E^k_{\ell} E^k_{\ell} = -\frac{1}{2}\int \mathrm{d}^3 r \varphi(\boldsymbol{r})\Delta\varphi(\boldsymbol{r}), \tag{17}$$

which yields the Coulomb interaction if one substitutes (16) into this equation. Thus by substituting (17) into (14), H' becomes identical with the Hamiltonian in the Coulomb gauge.

Summarizing the essentials of the transition from the temporal gauge into the Coulomb gauge, one learns that the use of the Hamilton formalism is basic for the success of this undertaking. Thus if we want to transfer this method into quantum theory we are forced to employ a combination of the Hamilton formalism with general algebraic representation theory of quantum fields.

3. Quantization in temporal gauge

We consider the system defined by the Lagrangian (1) in temporal gauge. In this case the canonical conjugate variables for A_k are given by (2), and due to the canonical quantization principle this leads to the commutation relations

$$\left[E^k(\boldsymbol{r},t), A^{k'}(\boldsymbol{r}',t)\right]_{-} = i\delta_{kk'}\delta(\boldsymbol{r}-\boldsymbol{r}') \tag{18}$$

for the electromagnetic fields, while for the fermion superspinors the anticommutation relations

$$\left[\psi_{\alpha\Lambda}(\boldsymbol{r},t)\psi_{\alpha'\Lambda'}(\boldsymbol{r}',t)\right]_{+} = (C\gamma^0)_{\alpha\alpha'}\sigma^1_{\Lambda\Lambda'}\delta(\boldsymbol{r}-\boldsymbol{r}') \tag{19}$$

have to hold. All other commutation relations for the electromagnetic fields and anticommutation relations for the Fermi fields vanish.

As a consequence of the canonical quantization principle also the Hamiltonian and the gauge transformations become operators, which however cannot be defined within a corresponding operator topology for infinite volume $V \equiv \mathbb{R}^3$. Nevertheless, in the following we shall use the Hamiltonian and the gauge operators as formal implementation operators on the field operator algebra which allows to apply the formalism of general quantum theory to the system under consideration. If afterwards this formalism is combined with the algebraic description of infinite systems this results in well-defined operators and operations in functional space which are equivalent to well-defined relations between well-defined matrix elements in physical state space. This has to be observed with respect to the following discussion.

As the residual gauge transformations are the source of constraint equations and redundant variables we have to study them in the quantum version of the theory.

If gauge transformations are to be implemented in a linear space (Hilbert space), they are to be subjected to the correspondence principle as any other transformation. That means one has to construct a linear

operator U, so that

$$\begin{aligned} \boldsymbol{A}' &= U\boldsymbol{A}U^{-1} = \boldsymbol{A} + \boldsymbol{\nabla}\omega, \\ \psi' &= U\psi U^{-1} = e^{-ie\omega\sigma^3}\psi, \\ \boldsymbol{E}' &= U\boldsymbol{E}U^{-1} = \boldsymbol{E}. \end{aligned} \tag{20}$$

The corresponding operator U reads for $\omega(\boldsymbol{r})$ testfunction $\in C_0^\infty(\mathbb{R}^3)$

$$U := \exp\left\{ -i \int \omega(\boldsymbol{r}) \left[\boldsymbol{\nabla} \cdot \boldsymbol{E}(\boldsymbol{r}) + \frac{ie}{2}\psi(\boldsymbol{r})C\gamma^0\sigma^2\psi(\boldsymbol{r}) \right] \mathrm{d}^3 r \right\} \tag{21}$$

and can be used in accordance with the comments given above. According to (20) the result of the application of U is analogous to the classical gauge transformation for the field operators. Therefore it follows that the Hamiltonian (5) is invariant under gauge transformations on the quantum level, too, i.e. we have

$$H' = UHU^{-1} = H \tag{22}$$

or $[H, U]_- = 0$. Thus H and U must have common eigenstates in a representation. However, as the residual gauge transformations are generated by means of infinitesimal generators

$$G(\boldsymbol{r}) := \boldsymbol{\nabla} \cdot \boldsymbol{E}(\boldsymbol{r}) - e\rho(\boldsymbol{r}) \tag{23}$$

with $\rho(\boldsymbol{r}) := -\frac{i}{2}\psi(\boldsymbol{r})C\gamma^0\sigma^2\psi(\boldsymbol{r})$, it follows from (22) that also

$$[H, G(\boldsymbol{r})]_- = 0 \tag{24}$$

must hold. Hence instead of U its generator can be used to characterize the eigenstates of the theory. Thus the complete state space of the theory is characterized by the eigenvalue equations

$$H|a\rangle = E_a|a\rangle, \tag{25}$$

$$G(\boldsymbol{r})|a\rangle = g_a|a\rangle, \tag{26}$$

if preliminarily we restrict the coordinate space to a finite volume V in order to obtain finite energy values. Later on we lift this restriction.

The solutions of these equations are not all physically relevant. According to the gauge principle all observable quantities have to be gauge invariant. If one extends this condition to the state space of a quantum theory the observable states $|a\rangle$ must be invariant under gauge transformations, i.e., among all possible representations of the gauge group only singlett states are physically meaningful. A singlett state is invariant under all group operations, i.e.

$$U|a\rangle_{\text{singl.}} = |a\rangle_{\text{singl.}}, \tag{27}$$

and from (27)

$$G(\boldsymbol{r})|a\rangle_{\text{singl.}} = 0 \tag{28}$$

can be deduced, or $g_a \equiv 0$ for physically relevant states. This is the quantum version of the Gauß law and due to the requirement of the invariance of the physical states $|a\rangle$ under gauge transformations it is obvious that in contrast to the equations of motion, the Gauß law cannot be assumed to hold as an operator equation. Rather from its derivation it must be considered as the consequence of the eigenvalue equation (27).

For an illustration of this condition one can compare (27) with a quantum mechanical system with rotational invariance of the Hamiltonian, where only states are admitted which are also rotationally invariant, i.e. have angular momentum zero. From these considerations it is also obvious that this interpretation of the Gauß law cannot be in conflict with the commutation relations (18) and one gets a physical motivation for the postulate of the weak Gauß law in conventional quantum electrodynamics which coincides with (28).

In quantum mechanics of systems with rotational invariance the states with definite value of angular momentum can be selected by the introduction of new variables which reflect the properties of angular momentum operators. In particular states with angular momentum zero are constructed by variables which are rotationally invariant. It is this fact which could induce one to use gauge invariant variables for a closer inspection of the gauge invariant states $|a\rangle$.

In accordance with algebraic representation theory of quantum states one has to assume that a special state space of a quantum field theory can be represented by a set of GNS-basis vectors. These basis vectors are generated by the projection of ordered monomials of field operators on a cyclic vacuum state $|0\rangle$ of the system. In the case of quantum electrodynamics the elementary independent field operators are given by

$\boldsymbol{A}$ and $\boldsymbol{E}$ and ψ and $\bar{\psi}$. Then the GNS-monomials are composed of these elementary operators which play the role of 'cartesian' coordinates for the field system. But following the idea of representing gauge invariant states by gauge invariant "coordinates", i.e. field variables, one can try to introduce combinations of elementary field operators which are gauge invariant; for instance

$$\left\{ \begin{array}{c} A_k^{\mathrm{tr}}(\boldsymbol{r}), E_k^{\mathrm{tr}}(\boldsymbol{r}), E_k^{\ell}(\boldsymbol{r}) \\ \psi_\beta^c(\boldsymbol{r})[\partial_k + ieA_k^{\ell}]\psi_\alpha(\boldsymbol{r}) \\ \psi_\beta(\boldsymbol{r})[\partial_k - ieA_k^{\ell}]\psi_\alpha^c(\boldsymbol{r}) \\ \psi_\alpha^c(\boldsymbol{r})\psi_\beta(\boldsymbol{r}),\ \psi_\alpha(\boldsymbol{r})\psi_\beta^c(\boldsymbol{r}) \end{array} \right\} \equiv \{\chi_Z(\boldsymbol{r})\}\,, \quad (29)$$

i.e. it holds

$$U\chi_Z(\boldsymbol{r})U^{-1} = \chi_Z(\boldsymbol{r}) \quad (30)$$

for all elements of the set (29) as can be verified by means of (20).

The corresponding GNS-states are then given by

$$|\boldsymbol{r}_1, Z_1 \ldots \boldsymbol{r}_N, Z_N\rangle := \left[\chi_{Z_1}(\boldsymbol{r}_1)\ldots\chi_{Z_N}(\boldsymbol{r}_N)\right]_{\mathrm{ord}}|0\rangle, \quad (31)$$

where $[]_{\mathrm{ord}}$ means an appropriate ordering which will be specified in the following section, and $|0\rangle$ means the gauge invariant cyclic vacuum state of the system. From these definitions it follows that the states (31) are gauge invariant, i.e.

$$U|\boldsymbol{r}_1, Z_1 \ldots \boldsymbol{r}_N, Z_N\rangle = |\boldsymbol{r}_1, Z_1 \ldots \boldsymbol{r}_N, Z_N\rangle, \quad (32)$$

and if we confine ourselves to small gauge transformations by using an expansion of (21) and substituting it into (32), then from (32) it follows

$$G(\boldsymbol{r})|\boldsymbol{r}_1, Z_1 \ldots \boldsymbol{r}_N, Z_N\rangle = 0, \quad (33)$$

i.e. for the gauge invariant GNS-basis states the postulate (28) is satisfied. Therefore, in accordance with the equivalent postulate of gauge invariant states $|a\rangle$ these states have to be represented by the general expansion

$$|a\rangle = \sum_N \int \sigma(\boldsymbol{r}_1, Z_1 \ldots \boldsymbol{r}_N, Z_N) \cdot |\boldsymbol{r}_1, Z_1 \ldots \boldsymbol{r}_N, Z_N\rangle \mathrm{d}^3 r_1 \ldots \mathrm{d}^3 r_N \quad (34)$$

which guarantees that postulate (28) is satisfied.

In the literature several attempts were made to reformulate abelian and nonabelian gauge theories in terms of gauge invariant local field operator combinations on the classical level [10]. But with respect to the use of such variables the difficulties arise on the quantum level. While in the abelian case, for instance quantum electrodynamics in Coulomb gauge, these difficulties can be mastered, in nonabelian theories no final answers allowing a straightforward algebraic treatment have so far been found. Hence we cannot apply this approach. A related approach is the use of nonlocal gauge invariant variables, i.e. the loop variables [11]. The corresponding formalism was successfully developed for quantum electrodynamics [12]. But in the same paper the unsolved problems of this approach were emphasized with respect to its applications to nonabelian theories, see also [13]. As we intend to treat nonabelian theories and consider the algebraic treatment of quantum electrodynamics only as a preparation for the nonabelian case, it therefore is of no use to apply loop variables. We thus continue to work with Cartesian coordinates in the following.

In the mathematical treatment of algebras the various representations of abstract algebras are characterized by the definition of "algebraic states", which are different from the physical states $|a\rangle$ in Hilbert space. In the subsequent section we will introduce a modified concept of algebraic states which allows the actual treatment of quantum systems by direct calculations. By means of this formalism we shall transform the temporal gauge into a form where the Coulomb forces explicitly appear.

4. Algebraic theory of quantum electrodynamics

Our aim is to derive a formulation of quantum electrodynamics which

i) is consistent with algebraic representation theory,

ii) allows practical calculations,

iii) contains no redundant variables, and

iv) can be transferred to nonabelian theory.

To achieve this aim we start with the algebraic formulation of quantum electrodynamics in temporal gauge and then extract the Coulomb force. We shall proceed in close analogy to the classical case and it is the first task in this section to show that the algebraic formalism indeed exhibits this analogy. In doing so

we cannot present in detail the foundations of this algebraic formalism; we refer to the literature [4] and only explain its essentials.

The basis of the algebraic evaluation of quantum field dynamics is given by the Heisenberg relation $i\dot{O} = [O, H]_-$ which when projected on energy eigenstates $\langle 0|$ and $|a\rangle$ yields

$$\Delta E \langle 0|O|a\rangle = \langle 0|[O, H]_-|a\rangle, \tag{35}$$

where O is any element of the field operator algebra but not H in general. This relation is only applicable to the genuine dynamical equations, but not to the constraints. In particular if we work in temporal gauge with the Gauß law as constraint, we have to include the Gauß law into this formalism.

Following previous investigations the algebra elements O can be chosen as monomials of field operators in order to describe the dynamics of the quantized fields. In our case these monomials are given by

$$e_m^n := \mathcal{A}\left(\psi_{I_1} \dots \psi_{I_n}\right)_t \mathcal{S}\left(B_{K_1} \dots B_{K_m}\right)_t, \tag{36}$$

where ψ_I are superspinors and B_K are superfields, $\psi := \{\psi, \psi^c\}$, $B := \{\boldsymbol{A}, \boldsymbol{E}\}$. The symbol $\mathcal{A}$ means antisymmetrization in the indices $I_1 \dots I_n$, while $\mathcal{S}$ means symmetrization in $K_1 \dots K_m$, and the index t indicates that all operators are taken at the same time t.

In the case of temporal gauge the $\boldsymbol{A}$- and $\boldsymbol{E}$-fields and the ψ- and ψ^c-fields are contained in (36) and these monomials serve as algebraic basis elements for the construction of GNS-basis states which can be alternatively defined by the set $\{\langle 0|e_m^n\}$, i.e. by the vectors

$$\begin{aligned} &\langle I_1 \dots I_n, K_1 \dots K_m| := \\ &\quad \langle 0|\mathcal{A}\left(\psi_{I_1} \dots \psi_{I_n}\right)_t \mathcal{S}\left(B_{K_1} \dots B_{K_m}\right)_t. \end{aligned} \tag{37}$$

Due to the quantization relations for field operators any disordered monomial can be expressed by ordered monomials, see [4]. Hence the GNS-states based on the ordered monomials are complete. If we consider a physical eigenstate $|a\rangle$ of the energy operator H, then this state can be characterized by the set of projections $\{\langle 0|e_m^n|a\rangle\}$ onto the basis vectors (37), and (35) can be used for the calculation of these projections as well as the corresponding energy differences $(E_a - E_0) \equiv \Delta E$ provided this calculation is related to a definite representation, see [4]. We remark that such energy differences can directly be derived and justified as Arveson spectrum from the Heisenberg automorphism in algebraic theory, see [1].

Furthermore, in order to obtain a compact formulation the set $\{\langle 0|e_m^n|a\rangle\}$ may be integrated into a generating functional state which includes the construction of a corresponding functional metric space and a biunique map between functional space and Hilbert space leading to functional quantum theory. In the framework of this functional quantum theory quantum electrodynamics can be formulated. We expound this formalism only so far as it is needed for the solution of our problem. For further details see [4].

The functional state corresponding to the state $|a\rangle$ is defined by

$$|\mathcal{G}(j, b, a)\rangle := \tag{38}$$

$$\sum_{n,m}^{\infty} \frac{i^n}{n!} \frac{i^m}{m!} \langle 0|\mathcal{A}\left(\psi_{I_1} \dots \psi_{I_n}\right)_t \mathcal{S}\left(B_{K_1} \dots B_{K_m}\right)_t |a\rangle \cdot j_{I_1} \dots j_{I_n} b_{K_1} \dots b_{K_m} |0\rangle_F,$$

where j_I and b_K are the functional source operators for fermions or bosons, respectively, while $|0\rangle_F$ is the functional Fock vacuum, i.e., the basis states in functional space are Fock states. However, we emphasize that the metric of this functional Fock space is *not* identical with the metric of the physical state space expressed in terms of projections $\{\langle 0|e_m^n|a\rangle\}$. This physical metric rather needs a special construction which we need not discuss here. For details see [4]. In this paper we only discuss a suitable functional formulation of (35). We apply the formal variables ψ_I and B_K for this derivation.

In this formal description the field equations (3) read

$$i\dot{\psi}_I = D_{II'}\psi_{I'} + W_{II'}^K B_K \psi_{I'}, \tag{39}$$

$$i\dot{B}_K = L_{KK'}B_{K'} + J_{II'}^K \psi_I \psi_{I'}, \tag{40}$$

and the Hamiltonian (5) is given by

$$\begin{aligned} H = &\frac{1}{2} A_{I_1 I} D_{I I_2} \psi_{I_1} \psi_{I_2} \\ &+ \frac{1}{2} A_{I_1 I} W_{I I_2}^K B_K \psi_{I_1} \psi_{I_2} \\ &+ \frac{1}{2} C_{K_1 K} L_{K K_2} B_{K_1} B_{K_2}, \end{aligned} \tag{41}$$

where we applied the following definitions

$$D_{II'} := -\left(i\alpha^k\partial_k - \beta m\right)_{\alpha\alpha'}\delta_{\lambda\lambda'}\delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{42}$$

$$W^K_{II'} := e\alpha^j_{\alpha\alpha'}\sigma^3_{\lambda\lambda'}\delta_{1\eta}\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(\boldsymbol{r}-\boldsymbol{z}), \tag{43}$$

$$L_{KK'} := i\delta_{jj'}\delta_{\eta 1}\delta_{2\eta'}\delta(\boldsymbol{z}-\boldsymbol{z}') + i\delta_{jj'}\delta_{\eta 2}\delta_{1\eta'}\Delta_{\boldsymbol{z}}P^{\boldsymbol{z}}_{\mathrm{tr}}\delta(\boldsymbol{z}-\boldsymbol{z}'), \tag{44}$$

$$J^K_{II'} := -\frac{e}{2}(C\gamma^k)_{\alpha\alpha'}\sigma^2_{\lambda\lambda'}\delta(\boldsymbol{z}-\boldsymbol{r})\delta(\boldsymbol{z}-\boldsymbol{r}')\delta_{2\eta}, \tag{45}$$

$$A_{II'} := \left(C\gamma^0\right)_{\alpha\alpha'}\sigma^1_{\lambda\lambda'}\delta(\boldsymbol{r}-\boldsymbol{r}'), \tag{46}$$

$$C_{KK'} := -\delta_{jj'}\sigma^2_{\eta\eta'}\delta(\boldsymbol{z}-\boldsymbol{z}'), \tag{47}$$

and where the indices I and K mean

$$I := \boldsymbol{r}, \alpha, \lambda;\ K := \boldsymbol{z}, j, \eta \tag{48}$$

with $\boldsymbol{r}$ = position, α = spinor index, λ = superspinor index, $\lambda = 1 \equiv$ spinor, $\lambda = 2 \equiv$ charge conjugated spinor, and $\boldsymbol{z}$ = position, j = vector index, $\eta = 1$ = vector potential, $\eta = 2$ = electric field.

In this form, by means of (35) - (38) the field equations (39), (40) can be mapped into functional space. We employ two theorems describing the rearrangement of unsymmetrical monomials of field operators into symmetrical monomials for boson operators and antisymmetrical ones for fermion operators and which are expressed in terms of generating functional states. For a proof of relations (49) we refer to [14] and [4]. The proof of (50) runs along the same lines as the proof of (49).

These theorems directly allow the mapping of (35) for all monomials O into functional space. In doing so, due to (49), (50) we obtain for any $\langle 0|(OH)|a\rangle$ and $\langle 0|(HO)|a\rangle$ a projection and if the set of O is complete this is equivalent to the functional equation

$$\left\{H\left(\frac{1}{i}\partial^b - \frac{i}{2}Cb, \frac{1}{i}\partial^f - \frac{i}{2}Aj\right) - H\left(\tfrac{1}{i}\partial^b + \tfrac{i}{2}Cb, \tfrac{1}{i}\partial^f + \tfrac{i}{2}Aj\right)\right\}|\mathcal{G}\rangle = \Delta E|\mathcal{G}\rangle, \tag{51}$$

where $H \equiv H(B,\psi)$ in accordance with (41).

This kind of transformation of Hilbert space equations into functional equations can be equally well applied to the Gauß constraint.

In our formal description the classical Gauß law reads

$$G^L(B,\psi) := N^L_K B_K + R^L_{I_1I_2}\psi_{I_1}\psi_{I_2} = 0 \tag{52}$$

with

$$N^L_{K_1} := \delta_{\eta_1 2}\partial^{\boldsymbol{r}}_{j_1}\delta(\boldsymbol{r}-\boldsymbol{r}_1)\,;\ \partial^{\boldsymbol{r}}_j := \frac{\partial}{\partial x^j}, \tag{53}$$

$$R^L_{I_1I_2} := -i\frac{e}{2}(C\gamma^0)_{\alpha_1\alpha_2}\sigma^2_{\lambda_1\lambda_2}\delta(\boldsymbol{r}-\boldsymbol{r}_1)\delta(\boldsymbol{r}-\boldsymbol{r}_2)$$

Proposition 1:

$$\langle 0|\mathcal{A}(\psi_{M_1}\dots\psi_{M_j})B_{V_1}\dots B_{V_\ell}\mathcal{A}(\psi_{I_1}\dots\psi_{I_n})\mathcal{S}(B_{K_1}\dots B_{K_m})|a\rangle = \frac{1}{i^{m+n}}F\langle 0|\partial^f_{I_n}\dots\partial^f_{I_1}\partial^b_{K_1}\dots\partial^b_{K_m}\left[\left(\frac{1}{i}\partial^f_{M_j}+\frac{i}{2}A_{M_jM'_j}j_{M'_j}\right)\dots\left(\frac{1}{i}\partial^f_{M_1}+\frac{i}{2}A_{M_1M'_1}j_{M'_1}\right)\right]_{\mathrm{as}} \cdot\left(\frac{1}{i}\partial^b_{V_\ell}+\frac{i}{2}C_{V_\ell V'_\ell}b_{V'_\ell}\right)\dots\left(\frac{1}{i}\partial^b_{V_1}+\frac{i}{2}C_{V_1V'_1}b_{V'_1}\right)|\mathcal{G}(j,b,a)\rangle \tag{49}$$

and

Proposition 2:

$$\langle 0|\mathcal{A}(\psi_{I_1}\dots\psi_{I_n})\mathcal{S}(B_{K_1}\dots B_{K_m})\mathcal{A}(\psi_{M_1}\dots\psi_{M_j})B_{V_1}\dots B_{V_\ell}|a\rangle = \frac{1}{i^{m+n}}F\langle 0|\partial^f_{I_n}\dots\partial^f_{I_1}\partial^b_{K_1}\dots\partial^b_{K_m}\left(\frac{1}{i}\partial^b_{V_1}-\frac{i}{2}C_{V_1V'_1}b_{V'_1}\right)\dots\left(\frac{1}{i}\partial^b_{V_\ell}-\frac{i}{2}C_{V_\ell V'_\ell}b_{V'_\ell}\right) \cdot\left[\left(\frac{1}{i}\partial^f_{M_j}-\frac{i}{2}A_{M_jM'_j}j_{M'_j}\right)\dots\left(\frac{1}{i}\partial^f_{M_1}-\frac{i}{2}A_{M_1M'_1}j_{M'_1}\right)\right]_{\mathrm{as}}|\mathcal{G}(j,b,a)\rangle, \tag{50}$$

where $\{\partial^f\}$ are the duals of $\{j\}$ and $\{\partial^b\}$ are the duals of $\{b\}$.

In quantum electrodynamics the Gauß law is weakly implemented by $G(B,\psi)|a\rangle = 0$ for all physical states $|a\rangle$, i.e. Equation (28).

In the algebraic version this weak condition is projected on the GNS-basis states, which leads to the set of equations

$$\langle 0|\mathcal{A}(\psi_{I_1}\dots\psi_{I_n})\mathcal{S}(B_{K_1}\dots B_{K_m})G^L(B,\psi)|a\rangle = 0 \quad (54)$$

for $(n, m = 0, 1\dots\infty)$.

Again we can apply Proposition 2, and in this way map conditions (54) into functional space. This yields

$$G^L\left(\frac{1}{i}\partial^b - \frac{i}{2}Cb, \frac{1}{i}\partial^f - \frac{i}{2}Aj\right)|\mathcal{G}\rangle = 0 \quad (55)$$

As H is invariant against U-transformations in Hilbert space, it follows that $G(\boldsymbol{r})$ commutes with H, i.e. equation (24).

If the map is isomorphic, then also the functional images of G^L and H have to fulfill these relations. In order to show this we study the commutation or anticommutation relations, respectively, of the functional variables occurring in equations (51) and (55).

The basic variables in functional space are the fermionic sources j_I and the bosonic sources b_K. Together with their duals ∂_I^f and ∂_K^b the nontrivial relations

$$\begin{aligned}\left[j_I, \partial^f_{I'}\right]_+ &= \delta_{II'}, \\ \left[\partial^b_{K'}, b_K\right]_- &= \delta_{KK'}\end{aligned} \quad (56)$$

hold, while all other commutators or anticommutators, respectively, vanish. The generating functionals (38) are embedded in a functional Fock space, i.e. $\partial_I^f|0\rangle_F = \partial_K^b|0\rangle_F = 0$, $\forall\ I, K$. As was already mentioned, this functional Fock space is only a book-keeping device and lacks any physical meaning. In particular this functional Fock space does not fix the field representations in various Hilbert spaces from the outset, because these representations are expressed by the matrix elements $\langle 0|O|a\rangle$ or $\langle 0|O|0\rangle$ themselves which are in no way influenced by this auxiliary Fock space. The only task of this Fock space is to allow a compact formulation of the field dynamics governed by the field algebra.

From (51) and (55) it is obvious that not the sources j and b and their duals themselves are the variables of the functional Hamilton and Gauß operators. Rather these operators exclusively depend on "Bogoljubov" transformed functional variables, which of course are also only book-keeping devices! For these new variables the nontrivial relations

$$\begin{aligned}&\left[\left(\frac{1}{i}\partial^b_K - \frac{i}{2}C_{KL}b_L\right), \left(\frac{1}{i}\partial^b_{K'} - \frac{i}{2}C_{K'L'}b_{L'}\right)\right]_- \\ &\quad = C_{KK'}\,\mathbb{I}, \\ &\left[\left(\frac{1}{i}\partial^b_K + \frac{i}{2}C_{KL}b_L\right), \left(\frac{1}{i}\partial^b_{K'} + \frac{i}{2}C_{K'L'}b_{L'}\right)\right]_- \\ &\quad = -C_{KK'}\,\mathbb{I},\end{aligned} \quad (57)$$

$$\begin{aligned}&\left[\left(\frac{1}{i}\partial^f_I - \frac{i}{2}A_{IJ}j_J\right), \left(\frac{1}{i}\partial^f_{I'} - \frac{i}{2}A_{I'J'}j_{J'}\right)\right]_+ \\ &\quad = -A_{II'}\,\mathbb{I}, \\ &\left[\left(\frac{1}{i}\partial^f_I + \frac{i}{2}A_{IJ}j_J\right), \left(\frac{1}{i}\partial^f_{I'} + \frac{i}{2}A_{I'J'}j_{J'}\right)\right]_+ \\ &\quad = A_{II'}\,\mathbb{I}\end{aligned} \quad (58)$$

hold, while *all* other commutators or anticommutators vanish.

We notice that in Hilbert space the quantization conditions (18), (19) can be rewritten in terms of the formal fields B_K and ψ_I which leads to

$$\begin{aligned}[\psi_I, \psi_{I'}]_+ &= A_{II'}\,\mathbb{I} \\ [B_K, B_{K'}]_- &= C_{KK'}\,\mathbb{I}\end{aligned} \quad (59)$$

A comparison with (57), (58) shows that the "Bogoljubov" transforms of j_I and b_K obey isomorphic commutation or anticommutation relations to their corresponding original fields, i.e., their inverse image in Hilbert space.

Proposition 3: The images of H and G in functional space obey the same commutation rules as the original operators in Hilbert space.

Proof: In accordance with (51) the functional Hamiltonian is defined by

$$\begin{aligned}\mathcal{H} := &H\left(\frac{1}{i}\partial^b - \frac{i}{2}Cb, \frac{1}{i}\partial^f - \frac{i}{2}Aj\right) \\ &- H\left(\frac{1}{i}\partial^b + \frac{i}{2}Cb, \frac{1}{i}\partial^f + \frac{i}{2}Aj\right),\end{aligned} \quad (60)$$

while the Gauß operators are defined by

$$\mathcal{G}^L := G^L\left(\frac{1}{i}\partial^b - \frac{i}{2}Cb, \frac{1}{i}\partial^f - \frac{i}{2}Aj\right). \quad (61)$$

Due to the vanishing of the commutators which are not mentioned in (57) and (58), it is

$$\left[\mathcal{G}^L, H\left(\frac{1}{i}\partial^b + \frac{i}{2}Cb, \frac{1}{i}\partial^f + \frac{i}{2}Aj\right)\right] \equiv 0. \quad (62)$$

In the remaining part of the $\mathcal{G}^L, \mathcal{H}$-commutator the variables are isomorphic to their inverse images in Hilbert space. Hence also the commutator is isomorphic to its inverse image in Hilbert space, i.e. it vanishes. ◇

Finally we observe an asymmetry with respect to the variables occurring in equations (51) and (55). This asymmetry is removed by the following theorem:

Proposition 4: Together with (55) also

$$G^L\left(\frac{1}{i}\partial^b + \frac{i}{2}Cb, \frac{1}{i}\partial^f + \frac{i}{2}Aj\right)|\mathcal{G}\rangle = 0 \quad (63)$$

holds as constraint in functional space.

Proof: We consider the matrix element

$$\langle 0|G^L\mathcal{A}\left(\psi_{I_1}\ldots\psi_{I_n}\right)_t\mathcal{S}\left(B_{K_1}\ldots B_{K_m}\right)_t|a\rangle \equiv \langle 0|G^L(B,\psi)e^n_m|a\rangle. \quad (64)$$

We decompose e^n_m into a gauge invariant part ϵ^n_m and a gauge dependent part κ^n_m, such that $e^n_m = \epsilon^n_m + \kappa^n_m$ holds. Then $\epsilon^n_m|a\rangle$ is a gauge invariant state and thus it is $G\epsilon^n_m|a\rangle = 0$. Hence we have

$$\langle 0|G^L e^n_m|a\rangle = \langle 0|G^L\kappa^n_m|a\rangle. \quad (65)$$

The complete set of states $|a\rangle$ is given by all eigenstates of Eqs. (25) and (26), i.e. with inclusion of all states for $g_a \neq 0$, which are gauge dependent. For this complete set $\{|b\rangle,\ \forall E_b, g_b\}$ the orthonormality relations $\langle b|b'\rangle = \delta_{bb'}$ hold. It is therefore possible to use the expansion

$$\langle 0|G^L\kappa^n_m = \sum_b c_b\langle b|. \quad (66)$$

We first show that the right-hand side of (66) cannot contain gauge invariant states. We decompose the set $\{|b\rangle, \forall E_b, g_b\}$ into two subsets $\{|b\rangle^g \forall E_b, g_b \neq 0\}\cup\{|b\rangle^0 \forall E_b, g_b \equiv 0\}$. As we work with the full set of cartesian field variables, hermitean conjugation does not alter the eigenvalues of the eigenvectors. Therefore we can rewrite expansion (66) in the form $\langle 0|G^L\kappa^n_m = \sum c_{bg}{}^g\langle b| + \sum c_{b0}{}^0\langle b|$, and if we multiply from the right by any $|b'\rangle^0$ this gives $\langle 0|G^L\kappa^n_m|b'\rangle^0 = c_{b'0}{}^0\langle b'|b'\rangle^0$. The right-hand side is invariant, the left-hand side is not. Hence all c_{b0} must vanish. This means that all states $\langle b|$ occurring in (66) must belong to eigenvalues $g_b \neq 0$. If we substitute (66) into (65) and observe the orthonormality relations we obtain

$$\langle 0|G^L e^n_m|a\rangle = \sum_b c_{bg}{}^g\langle b|a\rangle = 0 \quad (67)$$

as $|a\rangle$ belongs to the eigenvalue $g_a = 0$. Taking into account proposition 1 we obtain (63). ◇

Proposition 3 holds for G^L of (63), too.

5. Evaluation of Coulomb forces

We first summarize the results obtained by the algebraic formulation of quantum electrodynamics in functional space. In order to transform the Heisenberg dynamics into functional space in accordance with propositions 1 and 2 we have to perform the replacements

$$\begin{aligned}
\psi_\alpha(\boldsymbol{r}) &\Rightarrow \frac{1}{i}\partial_\alpha(\boldsymbol{r}) - \frac{i}{2}(C\gamma^0)_{\alpha\alpha'}j^c_{\alpha'}(\boldsymbol{r}), \quad (68)\\
\psi^c_\alpha(\boldsymbol{r}) &\Rightarrow \frac{1}{i}\partial^c_\alpha(\boldsymbol{r}) - \frac{i}{2}(C\gamma^0)_{\alpha\alpha'}j_{\alpha'}(\boldsymbol{r}),\\
A_j(\boldsymbol{r}) &\Rightarrow \frac{1}{i}\partial^A_j(\boldsymbol{r}) + \frac{1}{2}b^E_j(\boldsymbol{r}),\\
E_j(\boldsymbol{r}) &\Rightarrow \frac{1}{i}\partial^E_j(\boldsymbol{r}) - \frac{1}{2}b^A_j(\boldsymbol{r}),
\end{aligned}$$

for $i = 1$. These expressions fulfill the commutation and anticommutation rules (57) or (58), respectively.

Although these combinations follow directly from the rearrangement theorems it is convenient to introduce a slightly modified version in order to construct the transformed functional Fock spaces. So instead of the replacements for ψ^c and E we consider

$$\begin{aligned}
(C\gamma^0)_{\alpha\alpha'}\psi^c_{\alpha'}(\boldsymbol{r}) &= \psi^+_\alpha(\boldsymbol{r})\\
&\Rightarrow \frac{1}{i}(C\gamma^0)_{\alpha\alpha'}\partial^c_{\alpha'}(\boldsymbol{r}) - \frac{i}{2}j_\alpha(\boldsymbol{r}), \quad (69)\\
iE_j(\boldsymbol{r}) &\Rightarrow \partial^E_j(\boldsymbol{r}) - \tfrac{i}{2}b^A_j(\boldsymbol{r}),
\end{aligned}$$

while the other substitutions remain unchanged. Analogous relations hold for the $i = 2$ case. Thus in order to apply the rearrangement theorems we have to rewrite the Hamiltonian in terms of ψ, ψ^+, A, iE, and for these variables we substitute the new set of variables in functional space. For $i = 1$ we define

$$\begin{aligned} X_j^1(\boldsymbol{r}) &:= \frac{1}{i}\partial_j^A(\boldsymbol{r}) + \frac{1}{2}b_j^E(\boldsymbol{r}) =: X_K^1, \\ Y_j^1(\boldsymbol{r}) &:= \partial_j^E(\boldsymbol{r}) - \frac{i}{2}b_j^A(\boldsymbol{r}) =: Y_K^1, \\ \partial_\alpha^1(\boldsymbol{r}) &:= \frac{1}{i}\partial_\alpha(\boldsymbol{r}) - \frac{i}{2}(C\gamma^0)_{\alpha\beta}j_\beta^c(\boldsymbol{r}) =: \partial_I^1, \\ Z_\alpha^1(\boldsymbol{r}) &:= -\frac{1}{i}(C\gamma^0)_{\alpha\beta}\partial_\beta^c(\boldsymbol{r}) + \frac{i}{2}j_\alpha(\boldsymbol{r}) =: Z_I^1, \end{aligned} \tag{70}$$

and for $i = 2$ we define

$$\begin{aligned} X_j^2(\boldsymbol{r}) &:= \frac{1}{i}\partial_j^A(\boldsymbol{r}) - \frac{1}{2}b_j^E(\boldsymbol{r}) =: X_K^2, \\ Y_j^2(\boldsymbol{r}) &:= \partial_j^E(\boldsymbol{r}) + \frac{i}{2}b_j^A(\boldsymbol{r}) =: Y_K^2, \\ \partial_\alpha^2(\boldsymbol{r}) &:= \frac{1}{i}\partial_\alpha(\boldsymbol{r}) + \frac{i}{2}(C\gamma^0)_{\alpha\beta}j_\beta^c(\boldsymbol{r}) =: \partial_I^2, \\ Z_\alpha^2(\boldsymbol{r}) &:= \frac{1}{i}(C\gamma^0)_{\alpha\beta}\partial_\beta^c(\boldsymbol{r}) + \frac{i}{2}j_\alpha(\boldsymbol{r}) =: Z_I^2. \end{aligned} \tag{71}$$

These variables obey the commutation or anticommutation relations

$$\begin{aligned} \left[Y_K^1, X_{K'}^1\right]_- &= \delta_{KK'}, \\ \left[Z_I^1, \partial_{I'}^1\right]_+ &= \delta_{II'}, \\ \left[X_K^2, Y_{K'}^2\right]_- &= \delta_{KK'}, \\ \left[Z_I^2, \partial_{I'}^2\right]_+ &= \delta_{II'}, \end{aligned} \tag{72}$$

while all other commutators or anticommutators, respectively, vanish. The renormalized energy eigenvalue equation (51) in functional space then reads explicitly in these new variables

$$\begin{aligned} \sum_{i=1}^{2} \int \Bigg\{ &\frac{(-1)^i}{2}\left[Y_j^i(\boldsymbol{r})Y_j^i(\boldsymbol{r}) + X_j^{i,\mathrm{tr}}(\boldsymbol{r})\Delta X_j^{i,\mathrm{tr}}(\boldsymbol{r})\right] \\ &+ eX_j^i(\boldsymbol{r})Z_\alpha^i(\boldsymbol{r})\alpha_{\alpha\beta}^j\partial_\beta^i(\boldsymbol{r}) \\ - Z_\alpha^i(\boldsymbol{r})&\left(i\alpha^k\partial_k - \beta m\right)_{\alpha\beta}\partial_\beta^i(\boldsymbol{r})\Bigg\}\mathrm{d}^3r|\mathcal{G}\rangle = \Delta E|\mathcal{G}\rangle, \end{aligned} \tag{73}$$

and the Gauß constraints (55) and (63) are given by

$$\left\{\frac{1}{i}\partial_k Y_k^i(\boldsymbol{r}) + (-1)^i\frac{e}{2}\left[Z_\alpha^i(\boldsymbol{r})\partial_\alpha^i(\boldsymbol{r}) - \partial_\alpha^i(\boldsymbol{r})Z_\alpha^i(\boldsymbol{r})\right]\right\} \cdot |\mathcal{G}\rangle = 0, \quad i = 1, 2. \tag{74}$$

The doubling of the degrees of freedom is no failure of the theory, because X_K^1, X_K^2 and so forth result by a canonical transformation from the original source operators j^A, ∂^A and so forth, and this canonical transformation is biunique. In addition, the fact that two Hamiltonians occur in (73) indicates that the Heisenberg equation (35) is expressed in functional space and not the eigenvalue equation (25). Nevertheless, in the following we will show that this similarity of the functional equation to Hamiltonian eigenvalue equations allows a treatment of the quantum transformation which effects the explicit appearance of Coulomb forces in temporal gauge.

We observe that in the commutation relations (72) the sequence of the $i = 2$ variables is reversed in comparison with the $i = 1$ variables. In comparison with the conventional commutation relations in Fock space this difference enforces a different interpretation of the $i = 2$ variables compared to the $i = 1$ variables. In order to clearly identify the meaning of the various variables, in the following we use the notation

$$\begin{aligned} \hat{\partial}_K^1 &:= Y_K^1\,; \quad \hat{X}_K^1 := X_K^1, \\ \hat{\partial}_K^2 &:= X_K^2\,; \quad \hat{X}_K^2 := Y_K^2, \end{aligned} \tag{75}$$

which expresses the difference between these pairs of variables. However, the change of notation is not sufficient for a proper treatment of our transformation problem. Rather, together with this introduction of a new notation it is necessary to change the coordinate system in functional space in order to work successfully with the new variables.

In conventional many body physics the introduction of Bogoljubov transforms is coupled to the introduction of new vacua. It is likely to perform such a transition to a new vacuum also in the present case. Although the functional Fock space is only a bookkeeping device, a transformation of the Fock vacuum effects a rearrangement of the matrix elements in the generating functional state which is equivalent to a transformation of their defining state equations to a new set of variables. In this way the choice of a transformed functional vacuum is part of the strategy to obtain appropriate solutions of the functional equations under consideration and in an indirect way

it is connected with the fixing of a specific algebraic representation. If we denote the transformed functional vacuum by $|0\rangle'_F$ the operators $\hat{\partial}^1_K := Y^1_K$ and $\hat{\partial}^2_K := X^2_K$ should act as annihilation operators. This means

$$\hat{\partial}^1_K|0\rangle'_F = \left(\partial^E_K - \frac{i}{2}b^A_k\right)|0\rangle'_F \stackrel{!}{=} 0, \tag{76}$$

$$\hat{\partial}^2_K|0\rangle'_F = \frac{1}{i}\left(\partial^A_K - \frac{i}{2}b^E_k\right)|0\rangle'_F \stackrel{!}{=} 0. \tag{77}$$

Analogous conditions are imposed on the new fermion vacuum.

These conditions can be satisfied if $|0\rangle'_F$ is related to the original functional vacuum state $|0\rangle_F$ by

$$|0\rangle'_F = \exp\left\{\frac{i}{2}\int b^A_j(\boldsymbol{r})b^E_j(\boldsymbol{r})d^3r + \frac{1}{2}\int j^c_\alpha(\boldsymbol{r})(C\gamma^0)_{\alpha\beta}j_\beta(\boldsymbol{r})\mathrm{d}^3r\right\}|0\rangle_F. \tag{78}$$

So far we dealt with operator transformations. The simultaneous transformation to a new vacuum state (78) raises the question how this transformation affects the functional state vector $|\mathcal{G}\rangle$.

First of all one has to observe that the state vector $|\mathcal{G}\rangle$ transforms covariantly with respect to the transformation properties of its Hilbert space image $|a\rangle$, i.e., the original physical state vector. On the other hand, with respect to its Fock space representation in functional space the state $|\mathcal{G}\rangle$ has to be considered as an invariant vector. Writing (38) in symbolic form we have

$$|\mathcal{G}\rangle = \sum_n \tau_n|e_n\rangle_F, \tag{79}$$

where $\{|e_n\rangle_F\}$ is the set of Fock space basis vectors. If $|\mathcal{G}\rangle$ is referred to another set of basis vectors $\{|e_n\rangle'_F\}$, due to its invariance against changes of the basis system it is

$$|\mathcal{G}\rangle = \sum_n \tau_n|e_n\rangle_F = \sum_n \tau'_n|e_n\rangle'_F. \tag{80}$$

Hence we can equivalently expand the functional state vector with respect to the new basis $|0\rangle'_F$ and the new variables $\hat{X}^i_K$, $i = 1, 2$, which leads according to (76) (77) to a new Fock space construction in monomials of $\hat{X}^i_K$ applied to $|0\rangle'_F$.

In the following we assume $|\mathcal{G}\rangle$ to be expressed in this transformed Fock basis and solve (73) under this condition.

In a first step we rewrite the eigenvalue equation (73) and the constraints in the new variables. We define

$$\begin{aligned}\mathcal{H}_1 := &-\int\frac{1}{2}\left[\hat{\partial}^1_j(\boldsymbol{r})\hat{\partial}^1_j(\boldsymbol{r}) + \hat{X}^{1,\mathrm{tr}}_j(\boldsymbol{r})\Delta X^{1,\mathrm{tr}}_j(\boldsymbol{r})\right]\mathrm{d}^3r\\ &+\int e\hat{X}^1_j(\boldsymbol{r})Z^1_\alpha(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial^1_\beta(\boldsymbol{r})\mathrm{d}^3r\\ &-\int Z^1_\alpha(\boldsymbol{r})\left(i\alpha^k\partial_k - \beta m\right)_{\alpha\beta}\partial^1_\beta(\boldsymbol{r})\mathrm{d}^3r\end{aligned} \tag{81}$$

and

$$\begin{aligned}\mathcal{H}_2 := &\int\frac{1}{2}\left[\hat{X}^2_j(\boldsymbol{r})\hat{X}^2_j(\boldsymbol{r}) + \hat{\partial}^{2,\mathrm{tr}}_j(\boldsymbol{r})\Delta\hat{\partial}^{2,\mathrm{tr}}_j(\boldsymbol{r})\right]\mathrm{d}^3r\\ &+\int e\hat{\partial}^2_j(\boldsymbol{r})Z^2_\alpha(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial^2_\beta(\boldsymbol{r})\mathrm{d}^3r\\ &-\int Z^2_\alpha(\boldsymbol{r})\left(i\alpha^k\partial_k - \beta m\right)_{\alpha\beta}\partial^2_\beta(\boldsymbol{r})\mathrm{d}^3r,\end{aligned} \tag{82}$$

and thus from (73) we obtain the eigenvalue equation

$$(\mathcal{H}_1 + \mathcal{H}_2)\,|\mathcal{G}\rangle = \Delta E|\mathcal{G}\rangle, \tag{83}$$

while the Gauß constraints (74) go over into

$$\left[i\partial_k\hat{\partial}^1_k(\boldsymbol{r}) + \rho_1(\boldsymbol{r})\right]|\mathcal{G}\rangle = 0, \tag{84}$$

$$\left[i\partial_k\hat{X}^2_k(\boldsymbol{r}) - \rho_2(\boldsymbol{r})\right]|\mathcal{G}\rangle = 0, \tag{85}$$

$$\rho_i(\boldsymbol{r}) := \frac{e}{2}\left[Z^i_\alpha(\boldsymbol{r})\partial^i_\alpha(\boldsymbol{r}) - \partial^i_\alpha(\boldsymbol{r})Z^i_\alpha(\boldsymbol{r})\right]. \tag{86}$$

We observe that due to the transformation (78) and the corresponding commutation relations for the new variables, the operators $\hat{X}^i_K, Z^i_I$ are creation operators, while $\hat{\partial}^i_K, \partial^i_I$ are annihilation operators. Hence we have the relations

$$\begin{aligned}\hat{\partial}^i_K &\equiv \left(\hat{X}^i_K\right)^+\\ \partial^i_I &\equiv \left(Z^i_I\right)^+\end{aligned}, \quad i = 1, 2, \tag{87}$$

and (83), (84), (85) can be written in the form

$$\begin{aligned}&\left[\mathcal{H}_1\left((\hat{X}^1)^+, \hat{X}^1, (Z^1)^+, Z^1\right)\right.\\ &\left.+\mathcal{H}_2\left((\hat{X}^2)^+, \hat{X}^2, (Z^2)^+, Z^2\right)\right]|\mathcal{G}\rangle = \Delta E|\mathcal{G}\rangle,\end{aligned} \tag{88}$$

$$\left[i\partial_k\hat{X}^1_k(\boldsymbol{r})^+ + \rho_1(\boldsymbol{r})\right]|\mathcal{G}\rangle = 0, \tag{89}$$

$$\left[i\partial_k\hat{X}^2_k(\boldsymbol{r}) - \rho_2(\boldsymbol{r})\right]|\mathcal{G}\rangle = 0. \tag{90}$$

From these equations it follows that in (88) -(90) the two sets of variables $i = 1, 2$ are strictly separated and that in addition the Fock vacuum $|0\rangle'_F$ can be represented by $|0\rangle'_F = |\hat{0}\rangle^1_F \otimes |\hat{0}\rangle^2_F$, i.e. the direct product of the vacua for the two sets of variables. In accordance with these properties we can introduce the ansatz

$$|\mathcal{G}\rangle = |\mathcal{G}\rangle^1 \otimes |\mathcal{G}\rangle^2, \tag{91}$$

where the states $|\mathcal{G}\rangle^i$, $i = 1, 2$ are referred to the corresponding subspaces of the whole Fock space. These states then have to satisfy the equations

$$\begin{aligned} \mathcal{H}_1|\mathcal{G}\rangle^1 &= E_1|\mathcal{G}\rangle^1, \\ \mathcal{H}_2|\mathcal{G}\rangle^2 &= E_2|\mathcal{G}\rangle^2 \end{aligned} \tag{92}$$

and

$$\left[i\partial_k \hat{X}^1_k(\boldsymbol{r})^+ + \rho_1(\boldsymbol{r})\right]|\mathcal{G}\rangle^1 = 0, \tag{93}$$

$$\left[i\partial_k \hat{X}^2_k(\boldsymbol{r}) - \rho_2(\boldsymbol{r})\right]|\mathcal{G}\rangle^2 = 0. \tag{94}$$

Hence in the following we have to evaluate (92) - (94). As in these equations absolute energy values appear we consider the whole system enclosed in a finite volume V. Afterwards we remove this restriction and perform the limit $V \to \infty$. We consider the cases $i = 1, 2$ separately and later on combine the results.

We start with the $i = 1$ case. The divergence operation in (93) acts on the functional variables as a projection operator which annihilates the transversal parts of $\hat{\partial}^1_k(\boldsymbol{r}) \equiv \hat{X}^1_k(\boldsymbol{r})^+$. For a proper treament of (93) we thus have to decompose all vector variables in their longitudinal and transversal branches. This decomposition for functional variables can be performed in the same way as in function space and we obtain

$$\begin{aligned} \hat{\partial}^1_K = (P_\ell + P_t)\hat{\partial}^1_K &= \hat{\partial}^{1,\ell}_K + \hat{\partial}^{1,\mathrm{tr}}_K, \\ \hat{X}^1_K = (P_\ell + P_t)\hat{X}^1_K &= \hat{X}^{1,\ell}_K + \hat{X}^{1,\mathrm{tr}}_K \end{aligned} \tag{95}$$

with P_ℓ and P_t projectors in function space. With this decomposition the commutation relations (72) are transformed into

$$\left[\hat{\partial}^{1,\ell}_K, \hat{X}^{1,\ell}_{K'}\right]_- = P_\ell \delta_{KK'}, \tag{96}$$

$$\left[\hat{\partial}^{1,\mathrm{tr}}_K, \hat{X}^{1,\mathrm{tr}}_{K'}\right]_- = P_t \delta_{KK'}, \tag{97}$$

and with these variables (93) goes over into

$$\left[i\partial_k \hat{\partial}^{1,\ell}_k(\boldsymbol{r}) + \rho_1(\boldsymbol{r})\right]|\mathcal{G}\rangle^1 = 0. \tag{98}$$

In analogy to (15) and (16) we rewrite (98) in the form

$$\hat{\partial}^{1,\ell}_k(\boldsymbol{r})|\mathcal{G}\rangle^1 = -\frac{i}{4\pi}\int \partial_k C(\boldsymbol{r},\boldsymbol{r}')\rho_1(\boldsymbol{r}')\mathrm{d}^3r'|\mathcal{G}\rangle^1 \tag{99}$$

with $C(\boldsymbol{r},\boldsymbol{r}') := |\boldsymbol{r}-\boldsymbol{r}'|^{-1}$, and due to $\left[\hat{\partial}^{1,\ell}_k, \rho_1\right]_- = 0$ repeated application of (99) yields

$$\begin{aligned} &\int \hat{\partial}^{1,\ell}_k(\boldsymbol{r})\hat{\partial}^{1,\ell}_k(\boldsymbol{r})\mathrm{d}^3r|\mathcal{G}\rangle^1 = \\ &\quad -\tfrac{1}{4\pi}\int \rho_1(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_1(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r'|\mathcal{G}\rangle^1. \end{aligned} \tag{100}$$

Substituting this relation into (92) for $i = 1$ yields

$$\begin{aligned} &\Big\{\int \frac{1}{8\pi}\rho_1(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_1(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r' \\ &-\int\left[\frac{1}{2}\hat{\partial}^{1,\mathrm{tr}}_j(\boldsymbol{r})\hat{\partial}^{1,\mathrm{tr}}_j(\boldsymbol{r}) + \frac{1}{2}\hat{X}^{1,\mathrm{tr}}_j(\boldsymbol{r})\Delta\hat{X}^{1,\mathrm{tr}}_j(\boldsymbol{r})\right]\mathrm{d}^3r \\ &+\int e\left[\hat{X}^{1,\ell}_j(\boldsymbol{r}) + \hat{X}^{1,\mathrm{tr}}_j(\boldsymbol{r})\right]Z^1_\alpha(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial^1_\beta(\boldsymbol{r})\mathrm{d}^3r \\ &-\int\hat{Z}^1_\alpha(\boldsymbol{r})(i\alpha^k\partial_k - \beta m)_{\alpha\beta}\hat{\partial}^1_\beta(\boldsymbol{r})\mathrm{d}^3r\Big\}|\mathcal{G}\rangle^1 = E_1|\mathcal{G}\rangle^1. \end{aligned} \tag{101}$$

At this point the analogy to classical electrodynamics cannot be continued because it is not possible to remove the longitudinal $\hat{X}^{1,\ell}_K$ operators by a gauge transformation with classical function from (101). We are rather forced to take into account the properties of the generating functional states in the new variables. We know that by construction of the Fock space the basis vectors of the state $|\mathcal{G}\rangle^1$ must be monomials of $\hat{X}^1_K$ and Z^1_I applied to $|\hat{0}\rangle^1_F$. If we decompose $\hat{X}^1_K$ into longitudinal and transversal parts this decomposition allows a decomposition of $|\mathcal{G}\rangle^1$,

$$|\mathcal{G}\rangle^1 = |\mathcal{G}\rangle^1_c + |\mathcal{G}\rangle^1_r, \tag{102}$$

where $|\mathcal{G}\rangle^1_c$ contains only state vectors without monomials of $\hat{X}^{1,\ell}_K$. Substitution into (101) and comparison by powers of $\hat{X}^{1,\ell}_K$ leads to the zero order equation

$$\mathcal{H}^1_c|\mathcal{G}\rangle^1_c = E_1|\mathcal{G}\rangle^1_c. \tag{103}$$

This equation is identical with the $i = 1$ part of the functional equation of quantum electrodynamics in Coulomb gauge. Therefore the state functionals $|\mathcal{G}\rangle^1_c$ represent the physical part of the whole state $|\mathcal{G}\rangle^1$. In dependence on $|\mathcal{G}\rangle^1_c$ the part $|\mathcal{G}\rangle^1_r$ can be calculated

up to residual gauge freedoms, but is of no physical relevance.

We now discuss the $i = 2$ case. The corresponding equations are

$$\begin{aligned} &\mathcal{H}_2|\mathcal{G}\rangle^2 = E_2|\mathcal{G}\rangle^2, \\ &\left[i\partial_k \hat{X}^2_k(\boldsymbol{r}) - \rho_2(\boldsymbol{r})\right]|\mathcal{G}\rangle^2 = 0. \end{aligned} \tag{104}$$

In the functional subspace of the $i = 2$ variables these equations have nonhermitean operators. Nevertheless, their inverse images in Hilbert space are hermitean. Hence in accordance with the general theory of G.N.S. representations of hermitean operators [4] these operators possess left-hand solutions which belong to the same eigenvalues as the right-hand solutions. We consider the left-hand solutions ${}^2\langle\mathcal{S}|$ in functional space which are eigenstates of (104) in the form

$$\begin{aligned} &{}^2\langle\mathcal{S}|\mathcal{H}_2 = {}^2\langle\mathcal{S}|E_2, \\ &{}^2\langle\mathcal{S}|\left[i\partial_k \hat{X}^2_k(\boldsymbol{r}) - \rho_2(\boldsymbol{r})\right] = 0. \end{aligned} \tag{105}$$

By hermitean conjugation in functional space we obtain from (105)

$$\begin{aligned} &\mathcal{H}_2^+|\mathcal{S}\rangle^2 = E_2|\mathcal{S}\rangle^2, \\ &\left[-i\partial_k \hat{X}^2_k(\boldsymbol{r})^+ - \rho_2(\boldsymbol{r})^+\right]|\mathcal{S}\rangle^2 = 0, \end{aligned} \tag{106}$$

and if we take into account (86) and (82) and $\rho_2(\boldsymbol{r})^+ = \rho_2(\boldsymbol{r})$ we obtain for (106)

$$\begin{aligned} \mathcal{H}_2^+ = &\int \frac{1}{2}\left[\hat{\partial}^2_j(\boldsymbol{r})\hat{\partial}^2_j(\boldsymbol{r}) + \hat{X}^{2,\mathrm{tr}}_j(\boldsymbol{r})\Delta\hat{X}^{2,\mathrm{tr}}_j(\boldsymbol{r})\right]\mathrm{d}^3r \\ &+ \int e\hat{X}^2_j(\boldsymbol{r})Z^2_\alpha(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial^2_\beta(\boldsymbol{r})\mathrm{d}^3r \\ &- \int Z^2_\alpha(\boldsymbol{r})(i\alpha^k\partial_k - \beta m)_{\alpha\beta}\partial^2_\beta(\boldsymbol{r})\mathrm{d}^3r \end{aligned} \tag{107}$$

and

$$\left[-i\partial_k\partial^2_k(\boldsymbol{r}) - \rho_2(\boldsymbol{r})\right]|\mathcal{S}\rangle^2 = 0. \tag{108}$$

From (107) and (108) it follows that Eqs. (106) are analogous to the corresponding equations for the $i = 1$ case if one replaces $i = 1$ by $i = 2$. Therefore equations (106) can be treated along the same lines as for the $i = 1$ case. This yields

$$\mathcal{H}_c^{2+}|\mathcal{S}\rangle^2_c = E_2|\mathcal{S}^2\rangle_c \tag{109}$$

with

$$\begin{aligned} \mathcal{H}_c^{2+} := &\int -\frac{1}{8\pi}\rho_2(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_2(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r' \\ &+ \int \frac{1}{2}\left[\hat{\partial}^{2,\mathrm{tr}}_j(\boldsymbol{r})\hat{\partial}^{2,\mathrm{tr}}_j(\boldsymbol{r}) + \hat{X}^{2,\mathrm{tr}}_j(\boldsymbol{r})\Delta\hat{X}^{2,\mathrm{tr}}_j(\boldsymbol{r})\right]\mathrm{d}^3r \\ &+ \int e\hat{X}^{2,\mathrm{tr}}_j(\boldsymbol{r})Z^2_\alpha(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial^2_\beta(\boldsymbol{r})\mathrm{d}^3r \\ &- \int Z^2_\alpha(\boldsymbol{r})(i\alpha^k\partial_k - \beta m)_{\alpha\beta}\partial^2_\beta(\boldsymbol{r})\mathrm{d}^3r, \end{aligned} \tag{110}$$

which is identical with the Coulomb gauge Hamiltonian in functional space, apart from the special notation of the variables employed. Therefore also $\mathcal{H}_c^{2+}$ possesses a hermitean inverse image in Hilbert space, and thus with (109) the equations

$${}^2_c\langle\mathcal{S}|\mathcal{H}^2_c = {}^2_c\langle\mathcal{S}|E_2 \tag{111}$$

and

$$\mathcal{H}^2_c|\mathcal{G}\rangle^2_c = E_2|\mathcal{G}\rangle^2_c, \tag{112}$$

are simultaneously satisfied. Furthermore, if we now define the product state

$$|\mathcal{G}\rangle_c := |\mathcal{G}\rangle^1_c \otimes |\mathcal{G}\rangle^2_c \tag{113}$$

then this state satisfies the equation

$$\left(\mathcal{H}^2_c + \mathcal{H}^1_c\right)|\mathcal{G}\rangle_c = \Delta E|\mathcal{G}\rangle_c, \tag{114}$$

which is idential with the equation resulting from the direct quantization of electrodynamics in Coulomb gauge, as will be demonstrated in the following section.

6. Proof of equivalence

With (10), (14) - (17) the Hamiltonian in Coulomb gauge is given by

$$\begin{aligned} H^c := &\frac{1}{2}\int d^3r\left[E^{tr}_k E^{tr}_k + \left(\boldsymbol{\nabla}\times\boldsymbol{A}^{tr}\right)_k\left(\boldsymbol{\nabla}\times\boldsymbol{A}^{tr}\right)_k\right] \\ &-\frac{1}{2}\int \mathrm{d}^3r d^3r'\frac{e^2}{16\pi}\left(\psi C\gamma^0\sigma^2\psi\right)(\boldsymbol{r}) \\ &\qquad\cdot C(\boldsymbol{r},\boldsymbol{r}')\left(\psi C\gamma^0\sigma^2\psi\right)(\boldsymbol{r}') \\ &+\int \mathrm{d}^3r\psi^+\left(\frac{1}{i}\alpha_k\partial_k + m\beta\right)\psi \\ &-\int \mathrm{d}^3r\psi^+ e\alpha_k A^{tr}_k\psi \end{aligned} \tag{115}$$

In (115) we replace E^{tr}_k by $P_t\boldsymbol{E}$ and A^{tr}_k by $P_t\boldsymbol{A}$. Then we apply the formalism of sections 4 and 5 to the Coulomb gauge Hamiltonian (115). As due to

these replacements (115) depends on the the $\boldsymbol{E}$- and $\boldsymbol{A}$-fields, the calculation runs along the same lines as in the case of temporal gauge, apart from the elimination of the Gauß law. We obtain then the result that the Coulomb functional energy equation resulting from these calculations is equal to (114) if one identifies the functional state $|\mathcal{G}\rangle_c$ with the Coulomb gauge functional state. This simple proof of the equivalence of the temporal gauge functional equation and the Coulomb gauge functional equation rests on the use of the new variables (70) and (71). It is, however, of interest to compare our results with previous calculations performed in Coulomb gauge in order to gain confidence into the new formalism.

The algebraic treatment of quantum electrodynamics in Coulomb gauge without use of the new variables was performed in [14] and reviewed in [4]. There the following functional equation was obtained:

$$\begin{aligned}&\Big[j_I D_{II'}\partial_{I'} - ij_I W^K_{II'}\partial_{I'}\partial^b_K + b_K L_{KK'}\partial^b_{K'}\\&+ j_I C_I^{M_1M_2M_3}\Big(\partial_{M_1}\partial_{M_2}\partial_{M_3}\\&\qquad - \frac{1}{4}A_{M_2I_2}A_{M_1I_1}j_{I_2}j_{I_1}\partial_{M_3}\Big) \qquad (116)\\&+ iJ^K_{M_1M_2}b_K\Big(\partial_{M_1}\partial_{M_2} + \frac{1}{4}A_{I_1M_1}A_{I_2M_2}j_{I_1}j_{I_2}\Big)\Big]\\&\cdot|\mathcal{F}^c(j,b,a)\rangle = \Delta E|\mathcal{F}^c(j,b,a)\rangle\end{aligned}$$

with j_I fermion sources and ∂_I their duals, and b_K boson sources and ∂^b_K their duals. The symbols in (116) are defined as follows:

$$\begin{aligned}D_{II'} &:= -\left(i\alpha^k\partial_k - \beta m\right)_{\alpha\alpha'}\delta(\boldsymbol{r}-\boldsymbol{r}')\delta_{\lambda\lambda'},\\W^K_{II'} &:= e\alpha^k_{\alpha\alpha'}\delta(\boldsymbol{r}-\boldsymbol{r}')\delta(\boldsymbol{r}-\boldsymbol{z})\sigma^3_{\lambda\lambda''}\delta_{1\eta},\\L_{KK'} &:= i\delta_{jj'}\delta_{\eta1}\delta_{2\eta'}\delta(\boldsymbol{z}-\boldsymbol{z}')\\&\quad + i\Delta_{\boldsymbol{z}}\delta(\boldsymbol{z}-\boldsymbol{z}')\delta_{jj'}\delta_{\eta2}\delta_{1\eta'},\\C_I^{M_1M_2M_3} &:= -\frac{ie^2}{8\pi}\Big\{(C\gamma^0)_{\alpha_1\alpha_2}\delta_{\alpha\alpha_3}\sigma^2_{\lambda_1\lambda_2}\sigma^3_{\lambda\lambda_3}\\&\quad\cdot C(\boldsymbol{r}-\boldsymbol{r}_1)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\delta(\boldsymbol{r}_1-\boldsymbol{r}_3)\Big\}_{as(123)},\\J^K_{M_1M_2} &:= -\frac{e}{2}P^z_{tr}\delta(\boldsymbol{z}-\boldsymbol{r}_1)\delta(\boldsymbol{z}-\boldsymbol{r}_2)\\&\quad\cdot(C\gamma^k)_{\alpha_1\alpha_2}\sigma_{\lambda_1\lambda_2}\delta_{2\eta}.\end{aligned} \qquad (117)$$

For a comparison with (114) obtained by transformation from temporal gauge equations (73) and (74), equation (116) has to be further prepared.

Proposition 5: Equation (116) can be exclusively formulated in terms of transversal boson sources b^{tr}_K and their duals $\partial^{b,\mathrm{tr}}_K$.

Proof: Let $u_k(\boldsymbol{r})$ be an arbitrary vector field and $v_k(\boldsymbol{r})$ a transversal vector field $v_k(\boldsymbol{r}) \equiv v^{\mathrm{tr}}_k(\boldsymbol{r})$. Then

$$\int u_k(\boldsymbol{r})v^{\mathrm{tr}}_k(\boldsymbol{r})\mathrm{d}^3r = \int u^{\mathrm{tr}}_k(\boldsymbol{r})v^{\mathrm{tr}}_k(\boldsymbol{r})\mathrm{d}^3r \qquad (118)$$

provided the scalar product integral does exist. We apply this result to the state functional $|\mathcal{F}^c(j,b,a)\rangle$. In Coulomb gauge this functional state is exclusively formulated with respect to transversal $\boldsymbol{A}$- and $\boldsymbol{E}$-fields. Hence the corresponding matrix elements act as projectors on the corresponding sources, and due to (118) it must be

$$|\mathcal{F}^c(j,b,a)\rangle \equiv |\mathcal{F}^c(j,b^{\mathrm{tr}},a)\rangle, \qquad (119)$$

where b^{tr} is the transversal part of b. Furthermore, with $\partial^b_K = \left(\partial^{b,\ell}_K + \partial^{b,\mathrm{tr}}_K\right)$ we conclude

$$-ij_I W^K_{II'}\partial_{I'}\partial^b_K|\mathcal{F}^c\rangle = -ij_I W^K_{II'}\partial_{I'}\partial^{b,\mathrm{tr}}_K|\mathcal{F}^c\rangle \qquad (120)$$

as $\partial^{b\ell}_K|\mathcal{F}^c\rangle = 0$ due to (119). Observing the definition of $L_{KK'}$ and (118), we obtain

$$b_K L_{KK'}\partial^b_{K'}|\mathcal{F}^c\rangle = b^{\mathrm{tr}}_K L_{KK'}\partial^{b,\mathrm{tr}}_{K'}|\mathcal{F}^c\rangle. \qquad (121)$$

The transversal projector in $J^K_{M_1M_2}$ can be incorporated into b_K by replacing b_K by b^{tr}_K. Hence by taking into account these results, we have instead of (116) the equivalent equation

$$\begin{aligned}&\Big[j_I D_{II'}\partial_{I'} - ij_I W^K_{II'}\partial_{I'}\partial^{b,\mathrm{tr}}_K + b^{\mathrm{tr}}_K L_{KK'}\partial^{b,\mathrm{tr}}_{K'}\\&+ j_I C_I^{M_1M_2M_3}\Big(\partial_{M_1}\partial_{M_2}\partial_{M_3}\\&\qquad - \frac{1}{4}A_{M_2I_2}A_{M_1I_1}j_{I_2}j_{I_1}\partial_{M_3}\Big) \qquad (122)\\&+ i\hat{J}^K_{M_1M_2}b^{\mathrm{tr}}_K\Big(\partial_{M_1}\partial_{M_2} + \frac{1}{4}A_{I_1M_1}A_{I_2M_2}j_{I_1}j_{I_2}\Big)\Big]\\&\cdot|\mathcal{F}^c(j,^{\mathrm{tr}},a)\rangle = \Delta E|\mathcal{F}^c(j,b^{\mathrm{tr}},a)\rangle\end{aligned}$$

with

$$\hat{J}^K_{M_1M_2} := -\frac{e}{2}\delta(z-r_1)\delta(z-r_2) \cdot (C\gamma^k)_{\alpha_1\alpha_2}\sigma^2_{\lambda_1\lambda_2}\delta_{2\eta}, \tag{123}$$

i.e. all terms formulated in transversal boson variables. ◇

In a second step we replace the source operators b_K and their duals ∂^b_K in (116) or (122), respectively, by their definitions $b_K := \{b^E_k(\boldsymbol{r}), b^A_k(\boldsymbol{r})\}$ and $\partial_K := \{\partial^E_k(\boldsymbol{r}), \partial^A_k(\boldsymbol{r})\}$. In this notation (122) reads

$$\begin{aligned}
&\Big\{ i\int b^{A,\mathrm{tr}}_j(\boldsymbol{r})\partial^{E,\mathrm{tr}}_j(\boldsymbol{r})\mathrm{d}^3r + i\int b^{E,\mathrm{tr}}_j(\boldsymbol{r})\Delta\partial^{A,\mathrm{tr}}_j(\boldsymbol{r})\mathrm{d}^3r \\
&- i\frac{e}{2}b^{E,\mathrm{tr}}_j(\boldsymbol{r})\Big[\partial_{\alpha_1\lambda_1}(\boldsymbol{r})(C\gamma^j)_{\alpha_1\alpha_2}\sigma^2_{\lambda_1\lambda_2}\partial_{\alpha_2\lambda_2}(\boldsymbol{r}) \\
&\qquad - \frac{1}{4}j_{\beta_1\rho_1}(\boldsymbol{r})(\gamma^jC)_{\beta_1\beta_2}\sigma^2_{\rho_1\rho_2}j_{\beta_2\rho_2}(\boldsymbol{r})\Big]d^3r \\
&- \int j_{\alpha\lambda}(\boldsymbol{r})\left[i\alpha^k\partial_k - \beta m\right]_{\alpha\beta}\partial_{\beta\lambda}(\boldsymbol{r})\mathrm{d}^3r \\
&- ie\int j_{\alpha\lambda}(\boldsymbol{r})\alpha^j_{\alpha\alpha'}\sigma^3_{\lambda\lambda'}\partial_{\alpha'\lambda'}(\boldsymbol{r})\partial^{A,\mathrm{tr}}_j(\boldsymbol{r})\mathrm{d}^3r \\
&- \frac{ie^2}{8\pi}\int j_{\alpha\lambda}(\boldsymbol{r})\big[(C\gamma^0)_{\alpha_1\alpha_2}\delta_{\alpha\alpha_3}\sigma^2_{\lambda_1\lambda_2}\sigma^3_{\lambda\lambda_3} \\
&\qquad\qquad \cdot C(\boldsymbol{r},\boldsymbol{r}_1)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\delta(\boldsymbol{r}-\boldsymbol{r}_3)\big]_{as(123)} \\
&\qquad \cdot \big[\partial_{\alpha_1\lambda_1}(\boldsymbol{r}_1)\partial_{\alpha_2\lambda_2}(\boldsymbol{r}_2)\partial_{\alpha_3\lambda_3}(\boldsymbol{r}_3) \\
&\qquad\quad - \frac{1}{4}(C\gamma^0)_{\alpha_2\beta_2}\sigma^1_{\lambda_2\rho_2}(C\gamma^0)_{\alpha_1\beta_1}\sigma^1_{\lambda_1\rho_1} \\
&\qquad\qquad \cdot j_{\beta_2\lambda_2}(\boldsymbol{r}_2)j_{\beta_1\lambda_1}(\boldsymbol{r}_1)\partial_{\beta_3\lambda_3}(\boldsymbol{r}_3)\big] \\
&\qquad \cdot \mathrm{d}^3r\mathrm{d}^3r_1d^3r_2d^3r_3\Big\}|\mathcal{F}^c\rangle = \Delta E|\mathcal{F}^c\rangle.
\end{aligned} \tag{124}$$

The proof that the transformation of the temporal gauge yields the Coulomb gauge for the $|\mathcal{G}\rangle_c$ equation has to be performed by a comparison of (114) with (124). For convenience we explicitly write down (114):

Proposition 6: Equations (124) and (125) are equivalent.

Proof: The proof is rather extensive. We start with the discussion of the Coulomb terms in (125). We recall the definition of ρ_i given by (86) and the definitions of the corresponding variables (58) and (70), (71). Then for $i = 2$ we rewrite the Coulomb term in the following form

$$\begin{aligned}
&\frac{1}{8\pi}\int \rho_2(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_2(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r' \\
&= \frac{1}{4}C_{I_1I_2V_1V_2}\theta_{I_1}\theta_{I_2}\theta_{V_1}\theta_{V_2}
\end{aligned} \tag{126}$$

with

$$\theta_I := \left(\frac{1}{i}\partial_I + \frac{i}{2}A_{II'}j_{I'}\right), \tag{127}$$

$$C_{I_1I_2V_1V_2} := -\frac{e^2}{8\pi}\int \kappa_{I_1I_2}(\boldsymbol{r})\kappa_{V_1V_2}(\boldsymbol{r}')C(\boldsymbol{r},\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r', \tag{128}$$

and

$$\kappa_{I_1I_2}(\boldsymbol{r}) := (C\gamma^0)_{\alpha_1\alpha_2}\sigma^2_{\lambda_1\lambda_2}\delta(\boldsymbol{r}-\boldsymbol{r}_1)\delta(\boldsymbol{r}-\boldsymbol{r}_2), \tag{129}$$

while for $i = 1$ we write

$$\begin{aligned}
&\frac{1}{8\pi}\int \rho_1(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_1(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r' \\
&= \frac{1}{4}C_{I_1I_2V_1V_2}\phi_{I_1}\phi_{I_2}\phi_{V_1}\phi_{V_2}
\end{aligned} \tag{130}$$

with

$$\phi_I := \left(\frac{1}{i}\partial_I - \frac{i}{2}A_{II'}j_{I'}\right). \tag{131}$$

Definition (128) leads to the relations

$$\begin{aligned}
C_{I_1I_2V_1V_2} &= -C_{I_2I_1V_1V_2} = -C_{I_1I_2V_2V_1} \\
&= C_{V_1V_2I_1I_2}.
\end{aligned} \tag{132}$$

$$\begin{aligned}
&\Big\{\frac{1}{8\pi}\int \rho_1(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_1(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r' - \frac{1}{8\pi}\int \rho_2(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_2(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r' \\
&- \frac{1}{2}\int\left[\hat{\partial}^{1,\mathrm{tr}}_j(\boldsymbol{r})\hat{\partial}^{1,\mathrm{tr}}_j(\boldsymbol{r}) + \hat{X}^{1,\mathrm{tr}}_j(\boldsymbol{r})\Delta\hat{X}^{1,\mathrm{tr}}_j(\boldsymbol{r})\right]\mathrm{d}^3r + \frac{1}{2}\int\left[\hat{X}^{2,\mathrm{tr}}_j(\boldsymbol{r})\hat{X}^{2,\mathrm{tr}}_j(\boldsymbol{r}) + \hat{\partial}^{2,\mathrm{tr}}_j(\boldsymbol{r})\Delta\hat{\partial}^{2,\mathrm{tr}}_j(\boldsymbol{r})\right]\mathrm{d}^3r \\
&+ e\int\hat{X}^{1,\mathrm{tr}}_j(\boldsymbol{r})Z^1_\alpha(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial^1_\beta(\boldsymbol{r})\mathrm{d}^3r + e\int\hat{\partial}^{2,\mathrm{tr}}_j(\boldsymbol{r})Z^2_\alpha(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial^2_\beta(\boldsymbol{r})\mathrm{d}^3r - \sum_{i=1}^{2}Z^i_\alpha(\boldsymbol{r})(i\alpha^k\partial_k - \beta m)_{\alpha\beta}\partial^i_\beta(\boldsymbol{r})d^3r\Big\}|\mathcal{G}\rangle_c \\
&= \Delta E|\mathcal{G}\rangle_c
\end{aligned} \tag{125}$$

If we take into account these properties and the anti-commutation relations (58) we get

$$C_{I_1I_2I_3I_4}\theta_{I_1}\theta_{I_2}\theta_{I_3}\theta_{I_4} \tag{133}$$

$$= C_{[I_1I_2I_3I_4]_{\mathrm{as}(1234)}}\theta_{I_1}\theta_{I_2}\theta_{I_3}\theta_{I_4} + \frac{1}{2}C_{I_1I_2I_3I_4}A_{I_1I_4}A_{I_2I_3},$$

and this equation holds for the ϕ_I-variables, too, due to the invariance of (133) with respect to $A \to -A$. Hence we obtain

$$\sum_{i=1}^{2}(-1)^{i+1}\frac{1}{8\pi}\int \rho_i(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_i(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r' \tag{134}$$

$$= \frac{1}{4}C_{[I_1I_2I_3I_4]_{\mathrm{as}(1234)}}\left(\phi_{I_1}\phi_{I_2}\phi_{I_3}\phi_{I_4} - \theta_{I_1}\theta_{I_2}\theta_{I_3}\theta_{I_4}\right)$$

$$= j_I C_I^{I_1I_2I_3}\left(\partial_{I_1}\partial_{I_2}\partial_{I_3} + \frac{1}{4}A_{I_1I_1'}A_{I_2I_2'}j_{I_1'}j_{I_2'}\partial_{I_3}\right)$$

if we substitute (127) and (131) into this formula and directly evaluate it. The modified vertexfunction is given by

$$C_I^{I_1I_2I_3} := \frac{e^2}{8\pi}\int \kappa_{[I_1I_2}(\boldsymbol{r})\kappa_{I_3I_4]}(\boldsymbol{r}')_{\mathrm{as}(1234)} \cdot C(\boldsymbol{r},\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r'A_{I_4I}. \tag{135}$$

The functions κ are antisymmetric, i.e., $\kappa_{I_1I_2}(\boldsymbol{r}) = -\kappa_{I_2I_1}(\boldsymbol{r})$, and by means of this property the relation

$$\kappa_{[I_1I_2}(\boldsymbol{r})\kappa_{I_3I_4]}(\boldsymbol{r}')_{\mathrm{as}(1234)} = \kappa_{[I_1I_2}(\boldsymbol{r})\kappa_{I_3]I_4}(\boldsymbol{r}')_{\mathrm{as}(123)}. \tag{136}$$

follows. Therefore (135) can be written in the form

$$C_I^{I_1I_2I_3} = -\frac{ie^2}{8\pi}\left\{(C\gamma^0)_{\alpha_1\alpha_2}\delta_{\alpha\alpha_3}\sigma^2_{\lambda_1\lambda_2}\sigma^3_{\lambda\lambda_3} \cdot \delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\delta(\boldsymbol{r}-\boldsymbol{r}_3)C(\boldsymbol{r}-\boldsymbol{r}_1)\right\}_{\mathrm{as}(123)}. \tag{137}$$

Substitution of this expression into (134) and subsequent comparison of the right-hand side of (134) with the Coulomb part in (124) shows that the Coulomb parts of (124) and (125) are equal.

The remaining terms can be treated by substitution of the transformations (70), (71) into (125). In evaluating these expressions, commutation relations of the sources, properties of the Dirac algebra and partial integrations have to be applied in order to show the stated equivalence. In particular differentiations of source operators are shifted to their duals by partial integrations. One can easily make sure of the correctness of such operations by projection of the corresponding operators on the generating functional states. Partially the calculations are rather lengthy. Hence for brevity we can only give the results which are expressed by the correspondences of the various terms in (124) and (125). After a short calculation we obtain

$$-\frac{1}{2}\int \hat{\partial}_j^{1,\mathrm{tr}}(\boldsymbol{r})\hat{\partial}_j^{1,\mathrm{tr}}(\boldsymbol{r})\mathrm{d}^3r + \frac{1}{2}\int \hat{X}_j^{2,\mathrm{tr}}(\boldsymbol{r})\hat{X}_j^{2,\mathrm{tr}}(\boldsymbol{r})\mathrm{d}^3r \equiv i\int b_j^{A,\mathrm{tr}}(\boldsymbol{r})\partial_j^{E,\mathrm{tr}}(\boldsymbol{r})\mathrm{d}^3r \tag{138}$$

and

$$-\frac{1}{2}\int \hat{X}_j^{1,\mathrm{tr}}(\boldsymbol{r})\Delta\hat{X}_j^{1,\mathrm{tr}}(\boldsymbol{r})\mathrm{d}^3r + \frac{1}{2}\int \hat{\partial}_j^{2,\mathrm{tr}}(\boldsymbol{r})\Delta\hat{\partial}_j^{2,\mathrm{tr}}(\boldsymbol{r})\mathrm{d}^3r \equiv i\int b_j^{E,\mathrm{tr}}(\boldsymbol{r})\Delta\partial_j^{A,\mathrm{tr}}(\boldsymbol{r})\mathrm{d}^3r. \tag{139}$$

An extensive calculation is required to show that

$$e\int \hat{X}_j^{1,\mathrm{tr}}(\boldsymbol{r})Z_\alpha^1(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial_\beta^1(\boldsymbol{r})\mathrm{d}^3r + e\int \hat{\partial}_j^{2,\mathrm{tr}}(\boldsymbol{r})Z_\alpha^2(\boldsymbol{r})\alpha^j_{\alpha\beta}\partial_\beta^2(\boldsymbol{r})\mathrm{d}^3r \equiv e\int \frac{1}{i}j_{\alpha\lambda}(\boldsymbol{r})\alpha^j_{\alpha\alpha'}\sigma^3_{\lambda\lambda'}\partial_{\alpha'\lambda'}(\boldsymbol{r})\partial_j^{A,\mathrm{tr}}(\boldsymbol{r})\mathrm{d}^3r - i\frac{e}{2}\int b_j^{E,\mathrm{tr}}(\boldsymbol{r})\Big[\partial_{\alpha_1\lambda_1}(\boldsymbol{r})(C\gamma^j)_{\alpha_1\alpha_2}\sigma^2_{\lambda_1\lambda_2}\partial_{\alpha_2\lambda_2}(\boldsymbol{r}) - \frac{1}{4}j_{\beta_1\rho_1}(\boldsymbol{r})(\gamma^jC)_{\beta_1\beta_2}\sigma^2_{\rho_1\rho_2}j_{\beta_2\rho_2}(\boldsymbol{r})\Big]\mathrm{d}^3r \tag{140}$$

and

$$-\sum_{i=1}^{2}\int Z_\alpha^i(\boldsymbol{r})\left(i\alpha^k\partial_k - \beta m\right)_{\alpha\beta}\partial_\beta^i(\boldsymbol{r})\mathrm{d}^3r \equiv -\int j_{\alpha\lambda}(\boldsymbol{r})\left(i\alpha^k\partial_k - \beta m\right)_{\alpha\beta}\partial_{\beta\lambda}(\boldsymbol{r})\mathrm{d}^3r. \tag{141}$$

Finally from (134) and (137) it follows

$$\sum_{i=1}^{2}(-1)^{i+1}\frac{1}{8\pi}\int \rho_i(\boldsymbol{r})C(\boldsymbol{r},\boldsymbol{r}')\rho_i(\boldsymbol{r}')\mathrm{d}^3r\mathrm{d}^3r'$$
$$\equiv -\frac{ie^2}{8\pi}\int j_{\alpha\lambda}(\boldsymbol{r})\big[(C\gamma^0)_{\alpha_1\alpha_2}\delta_{\alpha\alpha_3}\sigma^2_{\lambda_1\lambda_2}\sigma^3_{\lambda\lambda_3}C(\boldsymbol{r},\boldsymbol{r}_1)$$
$$\cdot\,\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\delta(\boldsymbol{r}-\boldsymbol{r}_3)\big]_{as(123)}\big[\partial_{\alpha_1\lambda_1}(\boldsymbol{r}_1)\partial_{\alpha_2\lambda_2}(\boldsymbol{r}_2) \tag{142}$$
$$\cdot\,\partial_{\alpha_3\lambda_3}(\boldsymbol{r}_3)-\frac{1}{4}(C\gamma^0)_{\alpha_2\beta_2}\sigma^1_{\lambda_2\rho_2}(C\gamma^0)_{\alpha_1\beta_1}\sigma^1_{\lambda_1\rho_1}$$
$$\cdot\,j_{\beta_2\rho_2}(\boldsymbol{r}_2)j_{\beta_1\rho_1}(\boldsymbol{r}_1)\partial_{\alpha_3\lambda_3}(\boldsymbol{r}_3)\big]\,\mathrm{d}^3r_1\mathrm{d}^3r_2\mathrm{d}^3r_3\mathrm{d}^3r.$$

The set of calculated identities (138) - (142) means the identity of (124), (125) under transformations (70), (71). ◇

Summarizing these results, we have evaluated the Coulomb forces in temporal gauge. After this evaluation the functional energy equation in temporal gauge can be decomposed into a selfconsistent part identical with the functional energy equation in Coulomb gauge, and a part which contains the longitudinal fields. The latter is completely determined by the former and does not influence the energy eigenvalues. Hence after evaluation of the Coulomb forces in temporal gauge the relevant part of the temporal gauge energy equation is identical with the Coulomb gauge, although no Coulomb gauge condition is imposed on the system, i.e., we have reached our physical object to explicitly prepare the Coulomb forces and the mathematical object to obtain a selfconsistent calculation scheme. In addition, our proof demonstrates the equivalence of temporal gauge and Coulomb gauge in quantum electrodynamics based on the algebraic formalism. To complete the theory, of course, the equivalence of temporal gauge to covariant gauges has to be established and the relativistic invariance or covariance and the renormalizability has to be treated. But our primary aim was to develop an appropriate formalism for the resolution of quantum constraints. So the treatment of these problems remains to further papers. We only refer to [15] with respect to relativistic invariance of noncovariant gauges and to [16] with respect to renormalizability.

Finally a comment should be given about the algebraic treatment and the problem of the foundations of QED. As in QED renormalization leads to infinite constants, a rigorous mathematical justification seems not to be possible at present. This leads to doubts whether QED is correctly formulated at all, and to attempts to change the basic assumptions of QED, cf. [17]. However, with respect to the fact that so far the problem of inequivalent representations has not been sufficiently well treated it is too early to give up the basis of QED. It is this point of view which is pursued by the algebraic treatment although we do not use the usual covariant formalism. Apart from the resolution of constraints it is the advantage of the algebraic approach that it offers a clear concept how to introduce various representations into the representation-free Heisenberg dynamics, cf. [4]. And by treating global representations one is led to the introduction of connected VEV's which have to fulfill nonlinear dynamical equations. Only a study of the peculiarities and the solutions of such systems will give a profound insight in the presently postulated QED.

7. Functional canonical transformations

In the preceding sections we applied canonical transformations (automorphism) in functional space in order to resolve the quantum constraint equation, i.e. the Gauß law. Although these transformations primarily serve as a technical means, for a further evaluation of the theory their consequences have to be studied. In so doing it is not necessary to directly treat quantum electrodynamics. As the radiation field can be described by an ensemble of harmonic oscillators we can confine ourselves to the discussion of a single harmonic oscillator to demonstrate the meaning of these transformations. The only difference between the treatment of a single harmonic oscillator in functional space and that of an ensemble of harmonic oscillators, i.e. a quantum field theory in functional space, consists in the fact that the former canonical transformations of the oscillator go over into an automorphism of the algebra of functional sources for the ensemble. But this does not influence their general interpretation.

In the following we take over the results of the preceding sections without special reference, but we quote those formulas which are the corresponding electromagnetic expressions. We define the Hamiltonian of the oscillator by

$$H=\frac{1}{2}(p^2+q^2) \tag{143}$$

with the commutation relation, cf. (18),

$$[p, q]_- = -i\,. \tag{144}$$

With $Q_1 := q$ and $Q_2 := p$ the generating functional state is given by

$$|\mathcal{G}\rangle := \sum_{\ell=1}^{\infty} \frac{i^\ell}{\ell!} \langle 0|Q_{\eta_1} \ldots Q_{\eta_\ell}|n\rangle b_{\eta_1} \ldots b_{\eta_\ell}|0\rangle_F \tag{145}$$

for oscillator eigenstates $|n\rangle, n = 0, 1, 2 \ldots \infty$, and the eigenstate equation (35) is mapped into functional space by

$$(\mathcal{H}_1 - \mathcal{H}_2)|\mathcal{G}\rangle = \Delta E|\mathcal{G}\rangle \tag{146}$$

with

$$\mathcal{H}_1 = \frac{1}{2}\left(\frac{1}{i}\partial_2 - \frac{1}{2}b_1\right)^2 + \frac{1}{2}\left(\frac{1}{i}\partial_1 + \frac{1}{2}b_2\right)^2 \tag{147}$$

and

$$\mathcal{H}_2 = \frac{1}{2}\left(\frac{1}{i}\partial_2 + \frac{1}{2}b_1\right)^2 + \frac{1}{2}\left(\frac{1}{i}\partial_1 - \frac{1}{2}b_2\right)^2, \tag{148}$$

where $b_\eta, \partial_\eta, \eta = 1, 2$ are the source operators and their duals, i.e. $[\partial_\eta, b_\eta]_- = 1$. We introduce the new variables, cf. (70),

$$\begin{aligned} X_1 &= \frac{1}{i}\partial_1 + \frac{1}{2}b_2, \\ Y_1 &= \partial_2 - \frac{i}{2}b_1, \\ X_2 &= \frac{1}{i}\partial_1 - \frac{1}{2}b_2, \\ Y_2 &= \partial_2 + \frac{i}{2}b_1. \end{aligned} \tag{149}$$

These variables fulfill the commutation relations, cf. (72),

$$[X_\eta, Y_{\eta'}]_- = \delta_{\eta\eta'}(-1)^{\eta+1}\,,\ \eta, \eta' = 1, 2. \tag{150}$$

Thus X_1 corresponds to q, while Y_1 corresponds to p. Consequently we consider Y_1 as the dual of X_1 and write $\partial_{X_1} \equiv Y_1$. The reverse holds for X_2, Y_2. This leads to the following definitions, cf. (75),

$$\begin{aligned} \partial_{X_1} &\equiv \hat{\partial}_1 := Y_1\,;\ \hat{X}_1 := X_1, \\ \partial_{Y_2} &\equiv \hat{\partial}_2 := X_2\,;\ \hat{X}_2 := Y_2, \end{aligned} \tag{151}$$

and in these variables $\mathcal{H}_1 - \mathcal{H}_2$ can be written as

$$\mathcal{H}_1 - \mathcal{H}_2 = -\frac{1}{2}\left(\hat{\partial}_1^2 + \hat{\partial}_2^2\right) + \frac{1}{2}\left(\hat{X}_1^2 + \hat{X}_2^2\right), \tag{152}$$

while in the original sources we obtain

$$\mathcal{H}_1 - \mathcal{H}_2 = i(b_1\partial_2 - b_2\partial_1). \tag{153}$$

It should be noticed that q and p are hermitean operators while $b_\eta, \partial_\eta, \hat{X}_\eta, \hat{\partial}_{X_\eta}$ are not hermitean. Nevertheless this is no contradiction, as by these definitions of the auxiliary functional operators and corresponding states the map between algebraic state description in Hilbert space and functional space is biunique.

On the other hand, in Hilbert space the harmonic oscillator can be described by non-hermitean operators, too. By

$$\begin{aligned} q &= \frac{1}{\sqrt{2}}(a^+ + a), \\ p &= \frac{i}{\sqrt{2}}(a^+ - a) \end{aligned} \tag{154}$$

we can transform (143) into

$$H = a^+a + \frac{1}{2} \tag{155}$$

with $[aa^+]_- = 1$. Then the generating functional state can be defined by

$$|\tilde{\mathcal{G}}\rangle = \sum_{\ell=1}^{\infty} \frac{i^\ell}{\ell!} \langle 0|A_{\lambda_1} \ldots A_{\lambda_\ell}|n\rangle \tilde{b}_{\lambda_1} \ldots \tilde{b}_{\lambda_\ell}|\tilde{0}\rangle_F \tag{156}$$

with $A_1 := a^+$, $A_2 := a$, and the eigenstate equation (35) is mapped into functional space by

$$(\tilde{\mathcal{H}}_1 - \tilde{\mathcal{H}}_2)|\tilde{\mathcal{G}}\rangle = \Delta E|\tilde{\mathcal{G}}\rangle \tag{157}$$

with

$$\tilde{\mathcal{H}}_1 = \left(\frac{1}{i}\hat{\partial}_1 + \frac{i}{2}\hat{b}_2\right)\left(\frac{1}{i}\hat{\partial}_2 - \frac{i}{2}\hat{b}_1\right) + \frac{1}{2} \tag{158}$$

and

$$\tilde{\mathcal{H}}_2 = \left(\frac{1}{i}\hat{\partial}_2 + \frac{i}{2}\hat{b}_1\right)\left(\frac{1}{i}\hat{\partial}_1 - \frac{i}{2}\hat{b}_2\right) + \frac{1}{2}, \tag{159}$$

where $\tilde{b}_\eta, \tilde{\partial}_\eta, \eta = 1,2$ are the source operators and their duals, i.e. $[\tilde{\partial}_\eta, \tilde{b}_\eta]_- = 1$. We introduce the new variables

$$\begin{aligned}
\tilde{X}_1 &= -i\left(\tilde{\partial}_1 - \frac{1}{2}\tilde{b}_2\right), \qquad (160)\\
\tilde{Y}_1 &= -i\left(\tilde{\partial}_2 + \frac{1}{2}\tilde{b}_1\right),\\
\tilde{X}_2 &= -i\left(\tilde{\partial}_1 + \frac{1}{2}\tilde{b}_2\right),\\
\tilde{Y}_2 &= -i\left(\tilde{\partial}_2 - \frac{1}{2}\tilde{b}_1\right).
\end{aligned}$$

These variables fulfill the commutation relations

$$[\tilde{X}_\eta, \tilde{Y}_{\eta'}]_- = -\delta_{\eta\eta'}(-1)^{\eta+1}. \qquad (161)$$

Thus $\tilde{X}_1$ correspond to a^+, while $\tilde{Y}_1$ correspond to a. The reverse holds for $i = 2$. Hence we can write $\partial_{\tilde{X}_1} \equiv \tilde{Y}_1$ and $\partial_{\tilde{Y}_2} \equiv \tilde{X}_2$ and thus introduce the definitions

$$\begin{aligned}
\partial_{\tilde{X}_1} &\equiv \bar{\partial}_1 := \tilde{Y}_1\ ;\ \bar{X}_1 := \tilde{X}_1, \qquad (162)\\
\partial_{\tilde{Y}_2} &\equiv \bar{\partial}_2 := \tilde{X}_2\ ;\ \bar{X}_2 := \tilde{Y}_2.
\end{aligned}$$

Then $\tilde{\mathcal{H}}_1 - \tilde{\mathcal{H}}_2$ can be written as

$$\tilde{\mathcal{H}}_1 - \tilde{\mathcal{H}}_2 = \bar{X}_1\bar{\partial}_1 - \bar{X}_2\bar{\partial}_2, \qquad (163)$$

and in the original sources we obtain

$$\tilde{\mathcal{H}}_1 - \tilde{\mathcal{H}}_2 = -\left(\tilde{b}_1\tilde{\partial}_1 - \tilde{b}_2\tilde{\partial}_2\right). \qquad (164)$$

Next we consider the connection between $|\mathcal{G}\rangle$ and $|\tilde{\mathcal{G}}\rangle$. Equivalently to (145) we can represent $|\mathcal{G}\rangle$ by

$$|\mathcal{G}\rangle = \langle 0|e^{iqb_1+ipb_2}|n\rangle|0\rangle_F \qquad (165)$$

or by substitution of (154)

$$|\mathcal{G}\rangle = \langle 0|e^{ia^+\frac{1}{\sqrt{2}}(b_1+ib_2)+ia\frac{1}{\sqrt{2}}(b_1-ib_2)}|n\rangle|0\rangle_F. \qquad (166)$$

If $|\tilde{\mathcal{G}}\rangle$ is represented in the same way by

$$|\tilde{\mathcal{G}}\rangle = \langle 0|e^{ia^+\tilde{b}_1+ia\tilde{b}_2}|n\rangle|\tilde{0}\rangle_F \qquad (167)$$

it must be equal to (166), and with inclusion of hermitean conjugation a comparison yields

$$\begin{aligned}
\tilde{b}_1 &= \frac{1}{\sqrt{2}}(b_1 + ib_2)\ ;\ \tilde{\partial}_1 = \frac{1}{\sqrt{2}}(\partial_1 - i\partial_2), \qquad (168)\\
\tilde{b}_2 &= \frac{1}{\sqrt{2}}(b_1 - ib_2)\ ;\ \tilde{\partial}_2 = \frac{1}{\sqrt{2}}(\partial_1 + i\partial_2)
\end{aligned}$$

and $|\tilde{0}\rangle_F \equiv |0\rangle_F$ apart from an arbitrary phase factor.

With respect to the other transformations (149) (151) and (160) (162) we supply them by the postulate that the transformed space has to be a Fock space again, in order to maintain the representation as a generating functional state. This leads to the conditions

$$\begin{aligned}
\partial_\eta|0\rangle_F &\stackrel{!}{=} 0 \quad ;\ \eta = 1,2, \qquad (169)\\
\partial_{Y_2}|\hat{0}\rangle_F = \partial_{X_1}|\hat{0}\rangle_F &\stackrel{!}{=} 0,\\
\tilde{\partial}_\eta|\tilde{0}\rangle_F &\stackrel{!}{=} 0 \quad ;\ \eta = 1,2,\\
\partial_{\tilde{Y}_2}|\bar{0}\rangle_F = \hat{\partial}_{\tilde{X}_1}|\bar{0}\rangle_F &\stackrel{!}{=} 0,
\end{aligned}$$

where $|\hat{0}\rangle_F$ and $|\bar{0}\rangle_F$ are the new Fock vacua.

The simultaneous validity of all equations in (169) can only be achieved if the operator transformations are supplied by corresponding state transformations. We start with $\partial_\eta|0\rangle_F = 0$ which is guaranteed by direct Fock space construction. Then we define

$$|\hat{0}\rangle_F = e^{\frac{1}{2}ib_1b_2}|0\rangle_F \qquad (170)$$

and obtain

$$\partial_{X_1}|\hat{0}\rangle_F = \left(\partial_2 - \frac{i}{2}b_1\right)e^{\frac{1}{2}ib_1b_2}|0\rangle_F = 0, \qquad (171)$$

and with $X_2 = \partial_{Y_2}$

$$\partial_{Y_2}|\hat{0}\rangle_F = \left(\frac{1}{i}\partial_1 - \frac{1}{2}b_2\right)e^{\frac{1}{2}ib_1b_2}|0\rangle_F = 0. \qquad (172)$$

Thus the transformation (149) is accompanied by a state transformation (170) which can easily be extended to become a unitary transformation.

In the same way we can define

$$|\bar{0}\rangle_F = e^{-\frac{1}{2}\tilde{b}_1\tilde{b}_2}|\tilde{0}\rangle_F \qquad (173)$$

and satisfy the last line of (169) if $\tilde{\partial}_\eta|\tilde{0}\rangle_F = 0$ is satisfied. But this equation follows from (168) and $|\tilde{0}\rangle_F \equiv |0\rangle_F$.

Summarizing our results, we discussed the harmonic oscillator in the p, q-representation and the a^+, a-representation and their various accompanying representations in functional space. As the functional states of the harmonic oscillator can be exactly calculated, we can compare the various representations in order to find those coordinates which lead to the most simple analytic expressions of the functional states. For simplicity we confine ourselves to the discussion of the eigenstate $n = 1$, i.e. $|1\rangle \equiv a^+|0\rangle_H$, where $|0\rangle_H$ is the Hilbert space Fock vacuum.

First we consider $|\tilde{\mathcal{G}}\rangle$ defined by (156) or (167) respectively. If we observe $|\tilde{0}\rangle_F \equiv |0\rangle_F$ we obtain by normalordering for (167)

$$|\tilde{\mathcal{G}}\rangle = \langle 0|e^{ia^+\tilde{b}_1}e^{ia\tilde{b}_2}e^{-\frac{1}{2}\tilde{b}_1\tilde{b}_2}|n\rangle|0\rangle_F, \tag{174}$$

and with (173)

$$|\tilde{\mathcal{G}}\rangle = \langle 0|e^{ia^+\tilde{b}_1}e^{ia\tilde{b}_2}|n\rangle|\bar{0}\rangle_F = \langle 0|e^{ia\tilde{b}_2}|n\rangle|\bar{0}\rangle_F. \tag{175}$$

This yields for $n = 1$, i.e. $|\tilde{\mathcal{G}}(1)\rangle$, the state

$$|\tilde{\mathcal{G}}(1)\rangle = i\tilde{b}_2|\bar{0}\rangle_F, \tag{176}$$

and with (160) (162)

$$|\tilde{\mathcal{G}}(1)\rangle = (\tilde{X}_1 - \tilde{X}_2)|\bar{0}\rangle_F = \bar{X}_1|\bar{0}\rangle_F \tag{177}$$

i.e. a functional description which is as simple as in physical Fockspace and this is a solution of (157) in the representation (162). In addition this solution of (157) is also a solution of $\tilde{\mathcal{H}}_1$ and with corresponding variables of $\tilde{\mathcal{H}}_2$, respectively. Hence in this representation not only $\tilde{\mathcal{H}}_1 - \tilde{\mathcal{H}}_2$ but also $\tilde{\mathcal{H}}_1$ and $\tilde{\mathcal{H}}_2$ themselves have a physical meaning. Hence if one likes one can construct the solution of the renormalized energy equation (157) by $|\tilde{\mathcal{G}}\rangle = |\tilde{\mathcal{G}}\rangle_1 \otimes |\tilde{\mathcal{G}}\rangle_2$ where $|\tilde{\mathcal{G}}\rangle_i$ are solutions of

$$\tilde{\mathcal{H}}_i|\tilde{\mathcal{G}}\rangle_i = E_i|\tilde{\mathcal{G}}\rangle_i\,, \quad i = 1, 2\,. \tag{178}$$

We now consider $|\mathcal{G}\rangle$ defined by (145) or (165), respectively. We apply the equivalent formula (166) and obtain by normalordering

$$|\mathcal{G}\rangle = \langle 0|e^{ia\frac{1}{\sqrt{2}}(b_1+ib_2)}|n\rangle e^{\frac{1}{4}(b_1^2+b_2^2)}|0\rangle_F \tag{179}$$

and thus for $n = 1$

$$|\mathcal{G}(1)\rangle = \frac{i}{\sqrt{2}}(b_1 - ib_2)e^{\frac{1}{4}(b_1^2+b_2^2)-\frac{i}{2}b_1b_2}|\hat{0}\rangle_F, \tag{180}$$

With (149) and (151) this leads to the expression

$$\begin{aligned}|\mathcal{G}(1)\rangle = \frac{1}{\sqrt{2}}&\left[(\hat{X}_1 - \hat{\partial}_2) - (\hat{\partial}_1 - \hat{X}_2)\right]\\ &\cdot \exp\left[\frac{1}{4}(\hat{X}_1 - \hat{\partial}_2)^2 - \frac{1}{4}(\hat{\partial}_1 - \hat{X}_2)^2\right.\\ &\left.+\frac{1}{2}(\hat{\partial}_1 - \hat{X}_1)(\hat{X}_1 - \hat{\partial}_2)\right]|\hat{0}\rangle_F\end{aligned} \tag{181}$$

i.e., a rather complicated expression in contrast to (177). Hence, with respect to the solution of the renormalized energy equation these coordinates are unsuitable, and in these coordinates no meaningful interpretation of the energy equations for $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, can be given. On the other hand, just these coordinates have been used in the preceding sections for quantum electrodynamics and are very suitable for the resolution of the functional constraint equations. Therefore a solution of constraint equations and a solution of renormalized energy equations have to be done in different coordinate systems. It is the purpose of this section to demonstrate that these different coordinate systems are compatible, as they are connected by a canonical transformation which analogously can be applied in QED and QCD.

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